A Bibliography of Publications on the Numerical Calculation of \( \pi \)

Nelson H. F. Beebe  
University of Utah  
Department of Mathematics, 110 LCB  
155 S 1400 E RM 233  
Salt Lake City, UT 84112-0090  
USA  
Tel: +1 801 581 5254  
FAX: +1 801 581 4148  
E-mail: beebe@math.utah.edu, beebe@acm.org, beebe@computer.org (Internet)  
WWW URL: http://www.math.utah.edu/~beebe/

20 November 2018
Version 1.89

\[(\sin \alpha) / \alpha \] [127]. 0 [240]. 1 [253]. 1/\( \pi \) [275, 215, 216]. 1/\( \pi^2 \) [247, 216], 10,000 [57], 10,000,000 [152]. 16 [224]. 2 [60, 63]. 2 + 2 [239]. 2,000 [39]. 2,576,980,370,000 [241]. 29,360,000 [111]. \( \frac{\pi}{2} \) [251]. \( b \) [203]. \( C \) [292]. \( d \) [292]. \( e \) [112, 106, 64, 38, 125, 32, 39, 40, 239, 13, 62]. \( e^{-\pi/2} = i^2 \) [15]. \( \gamma \) [76]. \( GL(n, Z) \) [109]. \( N \) [128, 160, 95, 109, 151]. \( \phi \) [214, 221]. \( \pi \) [265, 138, 259, 294, 118, 205, 284, 70, 87, 210, 285, 279, 272, 132, 176, 128, 96, 226, 207, 14, 76, 295, 171, 165, 288, 153, 110, 154, 196, 258, 35, 111, 112, 28, 23, 193, 69, 77, 137, 160, 17, 106, 162, 91, 94, 100, 101, 250, 44, 64, 18, 214, 221, 222, 251, 209, 55, 149, 65, 38, 206, 37, 24, 131, 4, 261, 26, 21, 127, 5, 9, 10, 174, 223, 140, 145, 224, 114, 180, 115, 235, 121, 125, 236, 181, 92, 116, 163, 175, 71, 27, 22, 104, 133, 32, 39, 83, 225, 67]. \( \pi \) [47, 29, 189, 164, 200, 57, 48, 7, 146, 195, 40, 75, 19, 6, 58, 263, 68, 11, 12, 36, 239, 170, 241, 93, 61, 122, 30, 213, 129, 16, 13, 142, 152, 53, 185, 62, 8]. \( \pi, e \) [86, 105]. \( \pi/12 \) [31]. \( \pi/4 \) [46]. \( \pi/8 \) [31]. \( \pi = 2 \sum \arccot f_{2k+1} \) [78]. \( \pi^2 \) [249, 124, 48]. \( \pi^4 \) [103]. \( \pi \coth \pi \) [227]. \( q \) [235]. \( \sum 1/k^2 = \pi^2/6 \) [66]. \( \sum_{k=1}^\infty 1/k^2 = \pi^2/6 \) [54]. \( \sum_{k=1}^\infty \pi^2/6 \) [72]. \( \sum_{n=1}^\infty 1/n^2 = \pi^2/6 \) [107]. \( \sqrt{2} \) [86]. \( Z \) [109]. \( \zeta(2) = \pi^2/6 \) [274].

1975 [296]. 1983 [297].


3rd [298].

524 [79].

719 [136]. 786 [168].

'88 [299].

90 [143]. 90d [157]. 949 [288].


Historical [15, 11, 12]. History [69, 77, 283, 137, 115, 262]. Hold [191].
hyperbolic [9, 10]. Hypergeometric [275, 256].

ibid [76]. Identically [180]. Identities [166]. if [268, 277]. implementation
Integer [177, 126, 187, 172, 97, 120, 141]. integral [113]. Integrals [274].
In [295]. Interface [85]. interpretation [68]. introduction [228].
inverse [129]. inverse-tangent [129]. Involving [112, 16, 113]. Irrational

joy [162].

Kāshī [224]. Katahiro [164]. Key [191]. Know [277, 268]. Kochański
[261]. Kreis [30].

[238]. lost [288]. Lucas [140]. Lucky [138].

Machin [203, 221, 195, 129]. Machin-Type [203, 221]. Magical [281].
manuscript [131, 224, 122]. Many [154]. Math [199, 286, 76].
Mathematicians [49, 190]. Mathematics
[218, 204, 127, 303, 3, 25, 301, 197, 219, 228, 284]. May [191]. McKay [209].
Mean [110, 75, 193]. meaning [145]. mechanical [285]. mechanics [235].
Miraculous [144]. Mode [190]. Modular [98, 14, 157, 118, 209].
[89]. Multiple [73, 74, 79, 81, 85, 233, 168, 116, 241]. Multiple-Precision
[136, 143]. mysterious [207]. Mystery [281].

NATO [297]. Natural [150]. Newfoundland [297]. Newtonian [286]. no
nonextensive [235]. nonextensive-statistical-mechanics [235].
NORC [47]. Normal [192, 277, 194, 268, 270, 51, 33, 237, 52, 50].
Normality [259, 182]. Notable [293]. Note [33, 122, 2]. Notes [153, 15, 28, 84, 44, 107, 124, 55, 31, 24, 26, 103, 46, 175, 104, 54, 83, 72, 105, 48, 66].
Novel [244]. November [299]. Number [150, 206, 281, 269, 270, 115, 207, 8]. Numbers [266, 192, 201, 82, 109, 140, 194, 105, 135, 270, 51, 33, 173, 97, 120, 125, 181, 237, 151, 52, 50].
Numerical [112, 117, 158, 166, 6]. numerically [126].


REFERENCES


Xeon [295].

Year [147]. yields [128]. Youqin [165].


References


Shanks:1873:ENV


Polster:1879:NIS


vonLindemann:1882:ZGN

[8] Carl Louis Ferdinand von Lindemann. Über die Zahl \( \pi \). (German) [On the number \( \pi \)]. *Mathematische Annalen*, 20(??):213–225, ???? 1882. CODEN MAANA3. ISSN 0025-5831 (print), 1432-1807 (electronic). In this famous paper, von Lindemann proved that \( \pi \) is transcendental, showing that it is impossible to square the circle by compass and straightedge, a problem dating back before 400 BCE in Greece.

Glaisher:1883:CHL


Glaisher:1891:CHL


Smith:1895:HSA


Smith:1896:EHS


Veblen:1904:T


REFERENCES


12

REFERENCES


REFERENCES

Kazarinoff:1955:CNS


Nicolson:1955:SCN


Pennisi:1955:CNE


Felton:1957:ECM


Steinhaus:1958:PCB


Cassels:1959:PSA


Schmidt:1960:NN

Wrench:1960:EED


Matsuoka:1961:MNE


Dixon:1962:MNA

J. D. Dixon. Mathematical notes: $\pi$ is not algebraic of degree one or two. *American Mathematical Monthly*, 69(7):636, August/September 1962. CODEN AMMYAE. ISSN 0002-9890 (print), 1930-0972 (electronic).

Hardy:1962:CPS


Pathria:1962:SSR


Shanks:1962:CD

Daniel Shanks and John W. Wrench, Jr. Calculation of $\pi$ to 100,000 decimals. *Mathematics of Computation*, 16(77):76–99, January 1962. CODEN MCMPAF. ISSN 0025-5718 (print), 1088-6842 (electronic). URL http://www.jstor.org/stable/2003813. A note added in proof says: “J. M. Gerard of IBM United Kingdom Limited, who was then unaware of the computation described above, computed $\pi$ to 20,000 D on the 7090 in the London Data Centre on July 31, 1961. His program used Machin’s formula, (1) $[\pi = 16 \arctan(1/5) - 4 \arctan(1/239)]$, and required 39 minutes running time. His result agrees with ours to that number of decimals.”.

Esmenjaud-Bonnardel:1965:ESD

M. Esmenjaud-Bonnardel. Étude statistique des décimales de pi. (French) [Statistical study of the decimals of pi]. *Chiffres: Revue de l’Association
REFERENCES


[63] I. J. Good and T. N. Gover. The generalized serial test and the binary expansion of \( \sqrt{2} \). *Journal of the Royal Statistical Society, Series A (General)*, 131(??):434, ???? 1968. CODEN JSSAEF. ISSN 0035-9238. See [60].


REFERENCES


REFERENCES


REFERENCES


REFERENCES


REFERENCES


[107] Boo Rim Choe. Notes: An elementary proof of \( \sum_{n=1}^{\infty} 1/n^2 = \pi^2/6 \). *American Mathematical Monthly*, 94(7):662–663, August/September 1987. CODEN AMMYAE. ISSN 0002-9890 (print), 1930-0972 (electronic).


REFERENCES


Bailey:1989:NRR


Borwein:1989:RME


Chudnovsky:1989:CCC


Haastad:1989:PTA


Jochi:1989:CMA


Tee:1989:NBA


REFERENCES


[144] Steven Finch. The miraculous Bailey–Borwein–Plouffe pi algorithm. Recent URLs redirect to an unrelated site, but the one given here worked on
Hirata:1995:CTT


Rabinowitz:1995:SAD


Adamchik:1996:PYO


Barrow:1996:PSC


Dodge:1996:DSA


Dodge:1996:NRN


Plouffe:1996:CTD

REFERENCES


[153] Victor Adamchik and Stan Wagon. Notes: A simple formula for π. American Mathematical Monthly, 104(9):852–855, November 1997. CODEN AMMYAE. ISSN 0002-9890 (print), 1930-0972 (electronic). URL http://www.maa.org/pubs/monthly_nov97_toc.html. The authors employ Mathematica to extend earlier work of Bailey, Borwein [118], and Plouffe, [156], done in 1995, but only just published, that discovered an amazing formula for π as is a power series in 16−k, enabling any base-16 digit of π to be computed without knowledge of any prior digits. In this paper, Mathematica is used to find several simpler formulas having powers of 4−k. They also note that it has been proven that their methods cannot be used to exhibit similar formulas in powers of 10−k.


[157] D. H. Bailey, J. M. Borwein, and P. B. Borwein. Ramanujan, modular equations, and approximations to pi or How to compute one billion digits
REFERENCES


[160] Fabrice Bellard. A new formula to compute the n-th binary digit of \( \pi \). This formula is claimed in [240] to be somewhat faster to compute than the BBP formula., 1997. URL http://bellard.org/pi/pi_bin.pdf.


Tsaban:1998:RAP


Ferguson:1999:API


Gourdon:1999:PEU


Group:1999:P


Lange:1999:NEC


Lord:1999:RFA


Bailey:2000:IRD


Berggren:2000:PSB


Hardy:2000:CPS


Jaditz:2000:DPI


Kalantari:2000:NFA


Lagarias:2000:NAC


Percival:2000:PDE


Venkatachala:2000:RP


REFERENCES


Guillera:2006:NMO


Marsaglia:2006:RCS


Bailey:2007:EMA


Borwein:2008:CMD


Borwein:2008:VPG


Chan:2008:MTF


Chong:2008:EQ

REFERENCES


of Representatives—(1) supports the designation of a “Pi Day” and its celebration around the world; (2) recognizes the continuing importance of National Science Foundation’s math and science education programs; and (3) encourages schools and educators to observe the day with appropriate activities that teach students about Pi and engage them about the study of mathematics.”.


REFERENCES


REFERENCES


[249] David H. Bailey, Jonathan M. Borwein, Andrew Mattingly, and Glenn Wightwick. The computation of previously inaccessible digits of $\pi^2$ and Catalan’s constant. Report, Lawrence Berkeley National Laboratory; Centre for Computer Assisted Research Mathematics and its Applications (CARMA), University of Newcastle; IBM Australia, Berkeley, CA, USA; Callaghan, NSW 2308, Australia; St. Leonards, NSW 2065, Australia; Pyrmont, NSW 2009, Australia, April 11, 2011. 18 pp. URL http://crd.lbl.gov/~dhbailey/dhbpapers/bbp-bluegene.pdf.
REFERENCES

[250] D. Borwein and Jonathan M. Borwein. Proof of some experimentally conjectured formulas for $\pi$. Preprint, Department of Mathematics, University of Western Ontario and Centre for Computer-assisted Research Mathematics and its Applications (CARMA), School of Mathematical and Physical Sciences, University of Newcastle, London, ON, Canada and Callaghan, NSW 2308, Australia, December 4, 2011.


REFERENCES


REFERENCES


REFERENCES


[276] Alexander Yee and Shiguro Kondo. It stands at 10 trillion digits of pi... world record for both desktop and supercomputer!!! Web site, April 15, 2013. URL http://www.numberworld.org/y-crummer/. This site also
contains a table of digit records from 2009 to 2013 for various mathematical constants. The $\pi$ record of 10,000,000,000,050 decimal digits was reached on 17 October 2011 after 371 days of computation, and 45 hours of verification.


[279] Reinhard E. Ganz. The decimal expansion of $\pi$ is not statistically random. *Experimental mathematics*, 23(2):99–104, 2014. CODEN ????. ISSN 1058-6458 (print), 1944-950X (electronic). See the reproduction of results, and reanalysis, in [289], that reveals a flaw in the statistical analysis in this paper: Ganz used only a single blocksize in sampling digits, and that blocksize produces anomalous statistics.


REFERENCES

Borwein:2015:PPB


Cheng:2015:HBP


Friedmann:2015:QMD


Meyers:2015:NDP


Tracy:2015:OCC


Wardhaugh:2015:LCC

REFERENCES


REFERENCES


REFERENCES

Schumer:2004:MJ


Alladi:2013:RPW


Sidoli:2014:ATB


Pitici:2017:BWM