INTEGRAL ESTIMATION IN QUANTUM PHYSICS

by

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Doctor of Philosophy

in

Mathematical Physics

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STATEMENT OF DISSERTATION APPROVAL

The dissertation of   **Jane Doe**

has been approved by the following supervisory committee members:

Cornelius Lánczos ,	Chair(s)	**17 Feb 2016**
		Date Approved
Hans Bethe ,	Member	**17 Feb 2016**
		Date Approved
Niels Bohr ,	Member	**17 Feb 2016**
		Date Approved
Max Born ,	Member	**17 Feb 2016**
		Date Approved
Paul A. M. Dirac ,	Member	**17 Feb 2016**
		Date Approved

by **Petrus Marcus Aurelius Featherstone-Hough** , Chair/Dean of

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and by **Alice B. Toklas** , Dean of The Graduate School.

ABSTRACT

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

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For my parents, Alice and Bob.

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LIST OF FIGURES

LIST OF TABLES

NOTATION AND SYMBOLS

α	fine-structure (dimensionless) constant, approximately 1/137
α	radiation of doubly-ionized helium ions, He++
β	radiation of electrons
γ	radiation of very high frequency, beyond that of X rays
γ	Euler's constant, approximately $0.577\,215\ldots$
δ	stepsize in numerical integration
$\delta(x)$	Dirac's famous function
ϵ	a tiny number, usually in the context of a limit to zero
$\zeta(x)$	the famous Riemann zeta function
$\ldots$	$\ldots$
$\psi(x)$	logarithmic derivative of the gamma function
ω	frequency

TYPESETTING EXPERIMENTS

In this section, we use color in several places. The `\colorbox` command takes two arguments — a named color and text to be in black on a background of that color — and sets the text in a box with a small margin of width `\fboxsep` (set to `3.0pt` in this document).

Here, we want a tighter colored box that has a fixed height, and is independent of letter shape. We set the margin to zero inside a group so that the change is purely local, and so that height and depth of the line are not increased over what they would be if the colored box were not used. We prefix a TeX `\strut` to the user-supplied text, because that command expands to a zero-width box of the height and depth of parentheses, which, in most fonts, delimit the extent of letter shapes.

```
\newcommand {\hilitebox} [1] {{\fboxsep = 0pt\colorbox{pink}{\strut #1}}}
```

Here is a fragment from the first chapter in another thesis, set in *emphasized text* to distinguish it from the rest of this section:

In light of the known results, the consistency of empirical semivariogram and related estimators is widely considered a settled matter. For example, Lahiri, Lee, and Cressie [22] state:

> *The simpler and more commonly used nonparametric estimators of the variogram, such as the method of moments estimator of Matheron (1962) and its robustified versions due to Cressie and Hawkins (1980) have many desirable properties like, unbiasedness, consistency, etc. . . .*

Regarding a kernel estimator of the covariance function, Hall and Patil [14] remarked:

> *It is not difficult to see that if, as n increases, the points t_i become increasingly dense in each bounded subset of $\mathbb{R}^d$, then the bandwidth h may be chosen so that $\breve{\rho}(t) \to \rho(t)$ as $n \to \infty$, for each $t \in \mathbb{R}^d$.*

However, in order to be true, such statements would need to be qualified by many assumptions on the random field as well as on the observation locations. We will see in §2.3 that even for well-behaved random fields (e.g., ρ^*-mixing Gaussian random fields), it is not enough to assume that the observation locations become increasingly dense in each bounded subset; a stronger assumption

must be made to ensure that the observation locations do not become denser in one region too much faster than in others.

The text before the previous paragraph contained two `quote` environments separated by a line of prose. Here are some more tests of both kinds of LaTeX environments for showing text written by someone else.

This is a `quote` environment with one short line, following a fairly short paragraph of prose (in this, and following examples, the text is explicitly colored with a command like `\color{purple}` inside the environment before the text):

```
\begin{quote}
    \color{purple}
    14 March 2016 is $\pi \approx 3.1416$ day in funny notation.
    \hfill \emph{Web news reports}
\end{quote}
```

> 14 March 2016 is $\pi \approx 3.1416$ day in funny notation. *Web news reports*

This is a `quote` environment with three short lines, each a separate paragraph, following a fairly short paragraph of prose.

```
\begin{quote}
    \color{forestgreen}
    14 March 2016 is $\pi \approx 3.1416$ day in funny notation.
    \hfill \emph{Web news reports}

    14 March 2016 is $\pi \approx 3.1416$ day in funny notation.
    \hfill \emph{Web news reports}

    14 March 2016 is $\pi \approx 3.1416$ day in funny notation.
    \hfill \emph{Web news reports}
\end{quote}
```

> 14 March 2016 is $\pi \approx 3.1416$ day in funny notation. *Web news reports*
> 14 March 2016 is $\pi \approx 3.1416$ day in funny notation. *Web news reports*
> 14 March 2016 is $\pi \approx 3.1416$ day in funny notation. *Web news reports*

Here is another example, this time with separate colors for each paragraph:

```
\begin{quote}
```

```
    \color{darkkhaki}
    14 March 2016 is $\pi \approx 3.1416$ day in funny notation.
    \hfill \emph{Web news reports}

    \color{darkmagenta}
    14 March 2016 is $\pi \approx 3.1416$ day in funny notation.
    \hfill \emph{Web news reports}

    \color{darkcyan}
    14 March 2016 is $\pi \approx 3.1416$ day in funny notation.
    \hfill \emph{Web news reports}

    \color{darkorange}
    14 March 2016 is $\pi \approx 3.1416$ day in funny notation.
    14 March 2016 is $\pi \approx 3.1416$ day in funny notation.
    14 March 2016 is $\pi \approx 3.1416$ day in funny notation.
    \linebreak
    \strut
    \hfill \emph{Web news reports}
\end{quote}
```

> 14 March 2016 is $\pi \approx 3.1416$ day in funny notation. *Web news reports*
>
> 14 March 2016 is $\pi \approx 3.1416$ day in funny notation. *Web news reports*
>
> 14 March 2016 is $\pi \approx 3.1416$ day in funny notation. *Web news reports*
>
> 14 March 2016 is $\pi \approx 3.1416$ day in funny notation. 14 March 2016 is $\pi \approx 3.1416$ day in funny notation. 14 March 2016 is $\pi \approx 3.1416$ day in funny notation. *Web news reports*

Notice that `quote` paragraphs are *not* indented, but the environment itself *is* indented on the left and right by the value of `\leftmargin` (set to `27.37506pt` in this document, which should be identical to `2.5em`, where `1em` = `11.49739pt`).

For debugging purposes, we also have `\leftmargini` set to `27.37506pt`, and we have `\leftmarginii` set to `24.09003pt`.

This is a `quotation` environment with one paragraph, following a fairly short paragraph of prose (notice that the `quotation` paragraphs *are* indented):

```
\begin{quotation}
    \color{blue}
    Algebra is concerned with manipulation in
    \emph{time}, and geometry is concerned with
    \emph{space}. These are two orthogonal aspects
    of the world, and they represent two different
```

```
    points of view in mathematics.  Thus the
    argument or dialogue between mathematicians in
    the past about the relative importance of
    geometry and algebra represents something very
    fundamental.
    \hfill
    \emph{Sir Michael Atiyah}
    % Mathematics in the 20$^{th}$ century
    % NTM {\bf 10}(1--3) 25--39 (September 2002)
    % http://dx.doi.org/10.1007/BF03033096
\end{quotation}
```

> Algebra is concerned with manipulation in *time,* and geometry is concerned with *space.* These are two orthogonal aspects of the world, and they represent two different points of view in mathematics. Thus the argument or dialogue between mathematicians in the past about the relative importance of geometry and algebra represents something very fundamental. *Sir Michael Atiyah*

This is a `quotation` environment with three paragraphs, following a fairly short paragraph of prose:

> Algebra is concerned with manipulation in *time,* and geometry is concerned with *space.* These are two orthogonal aspects of the world, and they represent two different points of view in mathematics. Thus the argument or dialogue between mathematicians in the past about the relative importance of geometry and algebra represents something very fundamental. *Sir Michael Atiyah*
> Algebra is concerned with manipulation in *time,* and geometry is concerned with *space.* These are two orthogonal aspects of the world, and they represent two different points of view in mathematics. Thus the argument or dialogue between mathematicians in the past about the relative importance of geometry and algebra represents something very fundamental. *Sir Michael Atiyah*
> Algebra is concerned with manipulation in *time,* and geometry is concerned with *space.* These are two orthogonal aspects of the world, and they represent two different points of view in mathematics. Thus the argument or dialogue between mathematicians in the past about the relative importance of geometry and algebra represents something very fundamental. *Sir Michael Atiyah*

Now all following text should be back in double-spaced mode, and just go on and on

and on and on and on ....

Now all following text should be back in double-spaced mode, and just go on and on ....

CHAPTER 1

THE FIRST

This is a chapter. Remember that there should *always* be at least of few lines of prose after each sectional heading: failure to do so is a disservice to your readers, and also produces incorrect vertical spacing.

1.1 The first section

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

In **Figure 1.1** on the following page, we have a picture, and the LaTeX markup to include it looks like this:

```
\begin{figure}[t]
    \centerline{\includegraphics{fig1}}
    \caption{The first figure.}%
    \figlabel{fig1}
\end{figure}
```

We intentionally omitted an extension on the filename, so that this document can be processed with `latex` to get an output `.dvi` file, or with `pdflatex` to get an output `.pdf` file. The first case uses the file `fig1.eps`, and the second uses `fig1.pdf`. The `distill` or `ps2pdf` commands can be used to convert from *Encapulated PostScript* files to *Portable Document Format* files.

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Figure 1.1. The first figure.

1.1.1   The first subsection

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.1.2   The second subsection

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.1.3   The third subsection

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1.1.3.1   The first subsubsection

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.1.3.2   The second subsubsection

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.1.3.2.1   The first numbered paragraph   Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah

blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.1.3.2.2 **The second numbered paragraph** Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.2 The second section

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

In **Figure 1.2**, we have another picture.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Figure 1.2. The second figure.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

In Table 1.1, we show the 24-character lowercase Greek alphabet.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Table 1.1. Lowercase Greek letters.

α	alpha
β	beta
γ	gamma
δ	delta
ϵ, ε	epsilon
ζ	zeta
η	eta
θ, ϑ	theta
ι	iota
κ	kappa
λ	lambda
μ	mu
ν	nu
ξ	xi
o	omicron
π	pi
ρ	rho
σ, ς	sigma
τ	tau
υ	upsilon
ϕ, φ	phi
χ	chi
ψ	psi
ω	omega

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.3   The third section

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

In Table 1.2 on the next page, we show the 24-character uppercase Greek alphabet, 13 of which are identical with Latin letters, because the Romans borrowed several letters from the earlier Greek alphabet. However, the letter sounds do not always carry over: notice in particular the different names of the letter shapes **H** and **P**. In Modern Greek, β is pronounced *veeta*; the letter pair $\mu\pi$ is used to get a *bee* sound;

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Table 1.2. Uppercase Greek letters. Notice that several have the same letter shapes as Latin letters, and for those, TeX does not define macro names. For convenience, we supply our own definitions of these macros: `\Alpha`, `\Beta`, `\Epsilon`, `\Zeta`, `\Eta`, `\Iota`, `\Kappa`, `\Mu`, `\Nu`, `\Omicron`, `\Rho`, `\Tau`, and `\Chi`.

A	Alpha
B	Beta
Γ	Gamma
Δ	Delta
E	Epsilon
Z	Zeta
H	Eta
Θ	Theta
I	Iota
K	Kappa
Λ	Lambda
M	Mu
N	Nu
Ξ	Xi
O	Omicron
Π	Pi
P	Rho
Σ	Sigma
T	Tau
Υ	Upsilon
Φ	Phi
X	Chi
Ψ	Psi
Ω	Omega

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.4   Free software packages

The Free Software Foundation offers almost 300 software packages, most easily portable to many different operating systems and CPU platforms. They include at least these:

`a2ps`, `acct`, `acm`, `adns`, `alive`, `anubis`, `apl`, `archimedes`, `aris`, `aspell`, `auctex`, `autoconf-archive`, `autoconf`, `autogen`, `automake`, `avl`, `ballandpaddle`, `barcode`, `bash`, `bayonne`, `bc`, `binutils`, `bison`, `bool`, `bpel2owfn`, `c-graph`, `ccaudio`, `ccd2cue`, `ccrtp`,

ccscript, cfengine, cflow, cgicc, chess, cim, classpath, classpathx, clisp, combine, commoncpp, complexity, config, coreutils, cpio, cppi, cssc, cursynth, dap, datamash, ddd, ddrescue, dejagnu, denemo, dico, diction, diffutils, dionysus, direvent, dismal, dominion, easejs, ed, edma, electric, emacs, emms, enscript, fdisk, ferret, findutils, fisicalab, flex, fontutils, freedink, freefont, freeipmi, gama, garpd, gawk, gcal, gcc, gcide, gcl, gcompris, gdb, gdbm, gengen, gengetopt, gettext, gforth, ggradebook, ghostscript, gift, gleem, glibc, global, glpk, gmp, gnash, gnats, gnatsweb, gnu-c-manual, gnu-crypto, gnu-pw-mgr, gnubatch, gnubik, gnucap, gnucobol, gnudos, gnue, gnugo, gnuit, gnujump, gnukart, gnumach, gnun, gnunet, gnupod, gnuprologjava, gnuradio, gnurobots, gnuschool, gnushogi, gnusound, gnuspeech, gnuspool, gnustep, gnutls, gnutrition, gnuzilla, goptical, gperf, gprolog, greg, grep, groff, grub, gsasl, gsegrafix, gsl, gslip, gsrc, gss, gtypist, guile-gnome, guile-gtk, guile-ncurses, guile-opengl, guile-rpc, guile-sdl, guile, gv, gvpe, gxmessage, gzip, halifax, health, hello, help2man, hp2xx, httptunnel, hurd, hyperbole, idutils, ignuit, indent, inetutils, intlfonts, jacal, jel, jwhois, kawa, less, libcdio, libextractor, libffcall, libiconv, libidn, libmatheval, libmicrohttpd, librejs, libsigsegv, libtasn1, libtool, libunistring, libxmi, lightning, lilypond, liquidwar6, lsh, m4, macchanger, mailman, mailutils, make, marst, maverik, mc, mcron, mcsim, mdk, metahtml, mifluz, mig, miscfiles, mit-scheme, moe, motti, mpc, mpfr, mpria, mtools, myserver, nano, ncurses, nettle, non-gnu, ocrad, octave, oleo, orgadoc, osip, paperclips, parallel, parted, patch, pem, pexec, phantom, pies, plotutils, proxyknife, pspp, psychosynth, pth, pyconfigure, radius, rcs, readline, recutils, reftex, remotecontrol, rottlog, rpge, rush, sather, sauce, savannah, scm, screen, sed, serveez, sharutils, shishi, shmm, shtool, sipwitch, slib, smalltalk, solfege, spacechart, spell, sqltutor, src-highlite, stow, superopt, swbis, tar, termcap, termutils, teseq, teximpatient, texinfo, thales, time, tramp, trueprint, unifont, units, unrtf, userv, uucp, vc-dwim, vcdimager, vera, wb, wdiff, websocket4j, wget, which, windows, xaos, xboard, xhippo, xlogmaster, xnee, xorriso, and zile.

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In **Figure 1.3**, we have yet another picture.

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This is Figure 3

Figure 1.3. The third figure. This one has both short and long captions. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

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Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.5   Resizing figures

In **Figure 1.4** through **Figure 1.8** on the following page, we show how graphics files can be rescaled to convenient sizes, with input like this:

```
\begin{figure}[p]
    \centerline{\includegraphics[scale = 0.5]{fig1}}
    \caption{The fourth figure (at 50\% scale).}%
    \figlabel{fig4}
\end{figure}

\begin{figure}[p]
    \centerline{\includegraphics[scale = 0.75]{fig1}}
    \caption{The fifth figure (at 75\% scale).}%
    \figlabel{fig5}
\end{figure}
```

Figure 1.4. The fourth figure (at 50% scale).

Figure 1.5. The fifth figure (at 75% scale).

Figure 1.6. The sixth figure (at native size).

Figure 1.7. The seventh figure (at 125% scale).

Figure 1.8. The eighth figure (at 175% scale).

```
\begin{figure}[p]
    \centerline{\includegraphics{fig1}}
    \caption{The sixth figure (at native size).}%
    \figlabel{fig6}
\end{figure}

\begin{figure}[p]
    \centerline{\includegraphics[scale = 1.25]{fig1}}
    \caption{The seventh figure (at 125\% scale).}%
    \figlabel{fig7}
\end{figure}

\begin{figure}[p]
    \centerline{\includegraphics[scale = 1.75]{fig1}}
    \caption{The eighth figure (at 175\% scale).}%
    \figlabel{fig8}
\end{figure}
```

You can include multiple images, each with its own caption inside a single *unbreakable* figure environment, like this example shown in **Figure 1.9** and **Figure 1.10** on the next page, although you might want to adjust interfigure vertical space with a \vspace{} command:

```
\begin{figure}[t]
    \centerline{\includegraphics[scale = 0.5]{fig1}}
    \caption{The fourth figure (at 50\% scale).}%
    \figlabel{fig9}
    \vspace{3ex}
    \centerline{\includegraphics[scale = 0.75]{fig1}}
    \caption{The fifth figure (at 75\% scale).}%
    \figlabel{fig10}
\end{figure}
```

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This is Figure 1

Figure 1.9. The ninth figure (at 50% scale), boxed with the tenth figure.

This is Figure 1

Figure 1.10. The tenth figure (at 75% scale), boxed with the ninth figure.

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As a final example in this chapter, **Figure 1.11** on the following page shows how you can use LaTeX `picture` mode for annotating and positioning graphics images prepared outside LaTeX. The input that produced that figure looks like this:

```
\begin{figure}[t]
    %% The original image is 216bp wide by 72bp high, but we
    %% rescale it to 150 picture units divided by \unitlength:
    %% 150 / 0.75 = 112.5 mm
    \newcommand {\myfig} {\includegraphics[width = 112.5mm]{fig1}}

    \begin{center}
        %% The \unitlength is chosen to make the complete picture fit
        %% within the page margins

        \setlength{\unitlength}{0.75mm}

        %%%     insert (width,height)(lower-left-x,lower-left-y)
        \begin{picture}(170,70)(10,10)
            %% Place the included image FIRST!
            \put(10,10) {\myfig}

            %% Everything that follows OVERLAYS the original image!

            \graphpaper[10](0,0)(170,70)

            %% Mark the image center and corners by centered bullets
            \newcommand {\thedot} {\makebox (0,0) {$\bullet$}}
            \put( 85, 35) {\thedot}
            \put( 10, 10) {\thedot}
```

Figure 1.11. Using LaTeX `picture` mode for figure labeling and positioning.

```
        \put( 10, 60) {\thedot}
        \put(160, 10) {\thedot}
        \put(160, 60) {\thedot}

        \put( 10, 10) {\makebox (0,0) [r] {lower-left}}
        \put(160, 10) {\makebox (0,0) [l] {lower-right}}
        \put( 10, 60) {\makebox (0,0) [r] {upper-left}}
        \put(160, 60) {\makebox (0,0) [l] {upper-right}}
     \end{picture}
   \end{center}

   \vspace{2\baselineskip}

   \caption[Using \LaTeX{} \texttt{picture} mode]
        {Using \LaTeX{} \texttt{picture} mode for figure labeling
         and positioning.}
   \figlabel{picture-mode}
\end{figure}
```
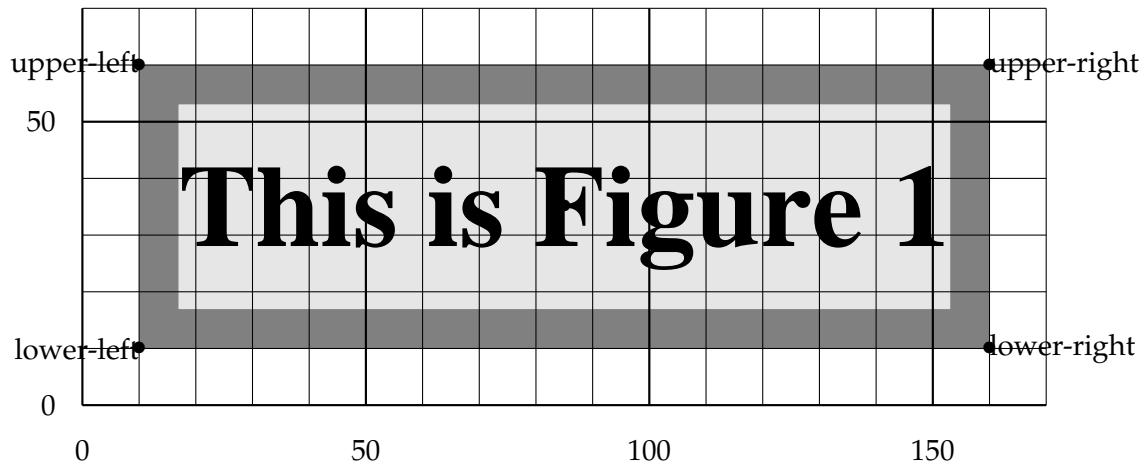
1.6   Summary and conclusions

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blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

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CHAPTER 2

THE SECOND

This is a chapter.

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CHAPTER 3

THE THIRD

This is a chapter.

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CHAPTER 4

THE FOURTH

This is a chapter.

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4.1   More on the topic

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Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

4.2   Even more on the topic

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah

blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

4.3   Summary and conclusions

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APPENDIX A

THE FIRST

This is an appendix. Notice that the LaTeX markup for an appendix is, surprisingly, `\chapter`. The `\appendix` command does not produce a heading; instead, it just changes the numbering style from numeric to alphabetic, and it changes the heading prefix from **CHAPTER** to **APPENDIX**.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah.

APPENDIX B

THE SECOND

This is an appendix.

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APPENDIX C

THE THIRD

This is an appendix.

There are several books [12, 19–21, 23–25, 27–30] listed in our bibliography.

We also reference several journal articles [1, 2, 4, 8–10, 13–18, 22, 31, 32] and three famous doctoral theses of later winners [3, 6, 7] of the Nobel Prize in Physics (1922, 1933, and 1921):

Notice that, even though those citations appeared in LaTeX `\cite{...}` commands with their BibTeX citation labels in reverse alphabetical order, thanks to the `citesort` package, their reference-list numbers have been sorted in numerically ascending order, and then range-reduced.

Mention should also be made of a famous Dutch computer scientist's first publication [5].

Font metrics are an important, albeit low-level, aspect of typesetting. See the *Adobe Systems* manual about that company's procedures [26].

The bibliography at the end of this thesis contains several examples of documents with non-English titles, and their BibTeX entries provide title translations following the practice recommended by the American Mathematical Society and SIAM. Here is a sample entry that shows how to do so:

```
@PhdThesis{Einstein:1905:NBM,
  author =       "Albert Einstein",
  title =        "{Eine Neue Bestimmung der Molek{\"u}ldimensionen}.
                 ({German}) [{A} new determination of molecular
                 dimensions]",
  type =         "Inaugural dissertation",
  school =       "Bern Wyss.",
  address =      "Bern, Switzerland",
  year =         "1905",
  bibdate =      "Fri Dec 17 10:46:57 2004",
```

```
bibsource =     "http://www.math.utah.edu/pub/tex/bib/einstein.bib",
note =          "Published in \cite{Einstein:1906:NBM}.",
acknowledgement = ack-nhfb,
language =      "German",
advisor =       "Alfred Kleiner (24 April 1849--3 July 1916)",
URL =           "http://en.wikipedia.org/wiki/Alfred_Kleiner",
remark =        "Received August 19, 1905 and published February 8,
                1906.",
Schilpp-number = "6",
}
```

The `note` field in that entry refers to another bibliography entry that need not have been directly cited in the document text. Such cross-references are common in BIBTEX files, especially for journal articles where there may be later comments and corrigenda that should be mentioned. Embedded `\cite{}` commands ensure that those possibly-important other entries are always included in the reference list when the entry is cited. The last bibliography entry [32] in this thesis has a long `note` field that tells more about what some may view as the most important paper in mathematics in the last century.

When entries cite other entries that cite other entries that cite other entries that …, multiple passes of LATEX and BIBTEX are needed to ensure consistency. That is another reason why document compilation should be guided by a `Makefile` or a batch script, rather than expecting the user to remember just how many passes are needed.

BIBTEX entries are *extensible*, in that arbitrary key/value pairs may be present that are not necessarily recognized by any bibliography style files. The `advisor`, `acknowledgement`, `bibdate`, `bibsource`, `language`, `remark`, and `Schilpp-number` fields are examples, and may be used by other software that processes BIBTEX entries, or by humans who read the entries. `DOI` and `URL` fields are currently recognized by only a few styles, but that situation will likely change as publishers demand that such important information be included in reference lists.

In BIBTEX `title` fields, braces protect words, such as proper nouns and acronyms, that cannot be downcased if the selected bibliography style would otherwise do so. In German, all nouns are capitalized, and the simple way to ensure their protection is to brace the entire German text in the title, as we did in the entry above.

The world's first significant computer program may have been that written in 1842

by Lady Augusta Ada Lovelace (1815–1852) for the computation of Bernoulli numbers [16, 18]. She was the assistant to Charles Babbage (1791–1871), and they are the world's first computer programmers. The programming language *Ada* is named after her, and is defined in the ANSI/MIL-STD-1815A Standard; its number commemorates the year of her birth.

We do not discuss mathematical *transforms* in this dissertation, but you can find that phrase in the index.

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