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The Effect of Finite Front Thickness on the Enhanced Speed of Propagation

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Abstract

Recently, Majda and Souganidis have presented a rigorous asymptotic theory governing the large-scale renormalized flame front dynamics for a convection-reaction-diffusion system involving KPP type chemistries and small scale turbulence. This theory is valid in the context of an infinitely thin reaction layer. Embid, Majda, and Souganidis have explored this rigorous theory within the context of a shear layer geometry, and demonstrate that the enhanced burning speed is sensitively dependent upon the presence of a mean wind transverse to the direction of the flame propagation. Here, we address the effect that a thin reaction layer may have on the enhanced flame propagation for the case of a small scale shear layer with and without a transverse mean wind. We show through high resolution, two dimensional numerical simulations that the enhanced burning speed is sensitively dependent on the presence of a transverse mean wind in a qualitatively similar fashion to the asymptotic theory; but further, we exhibit that a finite reaction zone yields effective burning speeds which are *smaller* than the theoretical predictions for infinitely thin flame fronts.

1 Introduction

An important problem from both the point of view of engineering applications as well as the theoretical understanding of the interaction of fluid motion with reacting chemistry concerns the study of turbulent combustion. The primary question regards how the presence of complex fluid motion may alter, modify, and enhance a chemical reaction to the advantage of increasing the mass consumption rate of reactants in practical applications. Naturally, such a question must depend upon both the nature of the turbulent fluid flow, as well as the particular chemistry considered. Nonetheless, certain phenomena may be expected qualitatively for a wide host of different chemistries and fluid flows. One example of such a phenomenon is the enhanced speed for premixed flame front propagation observed under a wide range of conditions, the turbulent burning velocity (Williams, 1985), which is defined by the average rate of propagation of the flame through the turbulent flow. The average propagation rate is seen to vary dramatically with the addition of a turbulent fluid motion through a complicated interaction occurring between the chemical reaction and the fluid turbulence. It goes without saying that a proper theoretical explanation of such a phenomena is extremely complicated due to the nonlinearities associated with the chemical reaction, and the nature of the fluid turbulence, in particular the many different scales involved.

At least in the flamelet regime, where the Damkohler number is quite large, there is a simplified model introduced by Damkohler (1940) that assumes the flame front is wrinkled by turbulence but propagates locally normal to itself with a laminar burning velocity. This is also known as Huygen's principle and the enhanced burning velocity can be viewed as the consequence of increased flame front surface area for reaction. In such a situation, the flame front propagation can be well described by the G-equation (Markstein, 1964) and there have been numerous studies using this approach to investigate the turbulent burning velocity (Ashurst, Kerstein and Williams, 1988; Yakhot, 1988; Sivashinsky, 1988; Ashurst and Sivashinsky, 1991; Zhu and Ronney, 1994).

However, the validity of utilizing Huygen's principle for the tracking of the large scale effective front position is certainly dependent upon both the type of chemistry considered as well as the size of the turbulent length scales relative to the reaction zone and the integral scale.

Much attention has been paid to the case of KPP combustion in the absence of a small scale fluid flow. (references?) In such a scenario, the chemical reaction is typically assumed to be a quadratic nonlinearity of a definite sign. Though there are natural physical concerns about the validity of such a chemistry; as a model, it does nicely exhibit many features common to a generic reaction-diffusion system. Recently, Majda and Souganidis have incorporated a small scale "turbulent" flow field into the dynamics for KPP combustion, and have derived rigorous effective front evolution equations through the use of nonlinear homogenized averaging (Majda and Souganidis, 1994). They study the following equation

$$\frac{\partial T}{\partial t} + \Big(\mathbf{V}(\mathbf{x},t) + \lambda \mathbf{v}(\epsilon^{-\alpha}\mathbf{x}, \epsilon^{-\alpha}t)\Big) \cdot \nabla T = \epsilon\bar{\kappa}\Delta T + \epsilon^{-1} f(T) \tag{1}$$

in the limit of vanishing ϵ for fixed values of the parameter α, satisfying $0 < \alpha < 1$. The nonlinearity is of KPP type, and typically is taken as $f(T) = KT(1-T)$, and the parameter λ, is a constant measuring the turbulent intensity. It is important to understand that such values of the parameter, α, at finite, but small values of ϵ, correspond to small scale turbulence which varies on length scales that are large relative to the width of the flame front but small in comparison with the integral scale set by the large scale flow, $\mathbf{V}(\mathbf{x},t)$. In general, they establish that in the limit of vanishing ϵ, the large scale, renormalized front dynamics are governed by a variational inequality as opposed to a more standard geometric front equation derived via the Huygen's principle. However, for the special case where the large scale mean flow field, $\mathbf{V}(\mathbf{x},t)$, is identically a constant vector, they have established that the renormalized, large scale front dynamics do evolve according to a geometric front equation with effective front speed given through an involved variational formula. It should be noted that the geometric front equation they derived has the general form

$$u_t = f(\nabla u) \tag{2}$$

which is different from the G-equation:

$$G_t + (\mathbf{V} + \lambda\mathbf{v}) \cdot \nabla G = S_L |\nabla G|, \tag{3}$$

where S_L is a prescribed laminar burning velocity. Eq. (2) is in fact a generalized Huygen's principle where the local burning speed has an orientation dependence.

More recently, in a series of papers, Embid, Majda, and Souganidis (1995a, 1995b, 1996) have explored the validity of Huygen's principle within the context of flows involving a large scale constant mean wind and a small scale "turbulent" flow by direct comparison of the rigorous nonlinear averaging of the full KPP convection-reaction-diffusion system and the nonlinear averaging of the G-equation. In the first paper of this series, the nonlinear averaging of the reaction-diffusion-advection system for a constant mean flow and simple small-scale shear layer was studied. For such a flow geometry, the variational formula yielding the effective front speed simplifies enormously and leads to explicit formulas. These formulas demonstrate interesting subtle dependence on the mean flow. Namely, provided the mean field exactly aligns with the shear direction, the direction of maximum enhanced flame speed aligns with the shear. Further, the enhanced speed of propagation in this direction is seen to be a linear function of the turbulent intensity λ. The situation is seen to be much different if the mean flow is orthogonal to the shear direction. In such a situation, the direction of maximal enhanced flame speed is seen to roughly align with the shear direction provided the transverse mean flow is sufficiently large. Moreover, the enhanced flame speed in this case depends quadratically upon the turbulent intensity for small values of λ, and the presence of a large transverse mean wind in fact reduces the enhanced burning speed. We remark that similar subtle dependence on the mean flow field has been seen in the entirely different context of the computation of enhanced diffusivities for passive transport (Majda and McLaughlin, 1993).

The second paper of the series (Embid, Majda, and Souganidis, 1995b) compares the averaging of the nonlinear reaction-diffusion-advection system with the homogenization of the G-equation. They demonstrate, for KPP chemistries, that the nonlinear averaging of the G-equation often underpredicts the turbulent burning velocity for many situations. The argument could be made that this is an immediate consequence of nature of the KPP chemistry, and does not occur with other more realistic modelings of chemical reactions.

However, in the most recent paper of the series (Embid, Majda and Sougadinis, 1996), it is demonstrated that this observation is not in fact an artifact of KPP type reactions, but occurs for more general reactions as well. They demonstrate this for a large class of piece-wise linear reactions introduced by Clavin and Linan (1983).

We remark that the theory of Majda and Souganidis, though rigorous, is derived in the context of infinitely thin flame fronts. Of course, any realistic chemical reaction will occur within a region of finite, non-zero volume. A natural question concerns the effect of finite, non-zero, flame front thickness on the effective front speed. The purpose of this paper is to explore this question through careful high resolution numerical computations of the full equation given in Eq. (1) in two space dimensions for finite, but small values of the parameter ϵ. We are primarily concerned with determining the effect that a finite reaction zone will have on the effective propagation rate as well as validating at finite ϵ the subtle mean flow dependence in the enhanced flame speed for the case of a turbulent shear layer. We will assess these issues by presenting a numerical comparison of solutions of the G-equation with the full KPP convection-reaction-diffusion system given in Eq. (1). The results of our computations explicitly demonstrate that these subtle effects are evident even for flames with finite thickness. Moreover, we will observe that this phenomenon is apparent with moderately large scale turbulence relative to the front thickness. Interestingly, we demonstrate that the primary effect of a reaction zone possessing finite, non-zero volume is to impede the propagation of the flame front *below* speeds predicted by the rigorous nonlinear averaging theory of Majda and Souganidis. The intuitive explanation for this observation is that a finite, non-zero reaction layer necessarily decreases the total surface area of the flame front, and by consequence must reduce the effective rate of propagation. We will demonstrate that this non-zero volume effect is most pronounced in shear layer geometries with large transverse mean winds. Finally, we will verify the following somewhat surprising result that the enhanced speed of propagation predicted by the G-equation with *moderately* large scale turbulence in fact overpredicts that observed in simulations of the full system in Eq. (1). We present convincing arguments that suggest that this observation is in no way caused by numerical artifacts by employing accurate error extrapolation based upon showing that the effect of finite mesh size on the full simulation of Eq. (1) in fact leads to an overestimation of the actual speed of enhanced propagation. We comment that it is the comparison of the *Homogenized* averaging of the G-equation with the rigorous nonlinear averaging of the full system in Eq. (1) which allowed Embid, Majda, and Souganidis (1995b) to concluded that the G equation underpredicts the actual enhanced propagation speed. In fact, we establish here that the effect of a finite front thickness is to reverse this ordering.

We remark that all of these observations should be quite useful in the construction of accurate Large Eddy Simulations and demonstrate that scheme coefficients should be chosen with care.

2 The Numerical Method

The numerical method we use here is based on the second-order finite difference method developed in the work of Xin and Zhu (1994). Here we briefly sketch the outline of the schemes used. The second-order finite difference method we use to approximate Eq. (1) is based on a Crank-Nicholson scheme where the linear convection is explicitly calculated at the half time steps by a second-order Godunov scheme. For discretization, a rectangular domain $[x_0, x_0+a]\times[y_0, y_0+b]$is partitioned into a collection of rectangular cells, where the cell centers are given by $\mathbf{x}_{i,j} = \left(x_0 + (i - \frac{1}{2})\Delta x, y_0 + (j - \frac{1}{2})\Delta y\right)$, where Δx and Δy are the cell size. At time $t = n\Delta t$, the numerical solution $T^n_{i,j}$ approximates the solution $T(\mathbf{x}_{i,j}, n\Delta t)$ and they satisfy the following discretized equations:

$$\frac{T^{n+1}_{i,j} - T^n_{i,j}}{\Delta t} + (\mathbf{v} \cdot \nabla T)^{n+\frac{1}{2}}_{i,j} = \left(L_h \left(\frac{T^n + T^{n+1}}{2}\right)\right)_{i,j} + \frac{f(T^n_{i,j}) + f(T^{n+1}_{i,j})}{2}, \qquad (4)$$

where L_h is the discretized Laplacian operator with the standard five point stencil. The discrete velocity field $\mathbf{v}$ combines the large scale mean flow and the small scale "turbulent" flow in the above equation. The calculation of $(\mathbf{v} \cdot \nabla T)^{n+\frac{1}{2}}$ is based on the method introduced by Colella (1990) which is second-order accurate for smooth solutions and stable in regions with steep gradients. The resulting nonlinear system for T^{n+1} has the following form:

$$T^{n+1}_{i,j} - \frac{\Delta t}{2}\left[(L_h T^{n+1})_{i,j} + f(T^{n+1}_{i,j})\right] = F(T^n)_{i,j}, \qquad (5)$$

with the given forcing term evaluated at $t = n\Delta t$,

$$F(T^n)_{i,j} = T^n_{i,j} + \frac{\Delta t}{2}\left[(L_h T^n)_{i,j} + f(T^n_{i,j})\right] - \Delta t(\mathbf{v} \cdot \nabla T)^{n+\frac{1}{2}}_{i,j} \ . \qquad (6)$$

This nonlinear system is solved by a package NKSOL which uses a GMRES iterative scheme. The stability condition is only required for the explicit calculation of the forcing term $(\mathbf{v} \cdot \nabla T)^{n+\frac{1}{2}}$ and this CFL condition is given by

$$\Delta t \leq \min\left(\frac{h}{|v_{1,i+\frac{1}{2},j}|}, \frac{h}{|v_{2,i,j+\frac{1}{2}}|}\right) \ , \qquad (7)$$

where v_1 and v_2 are the x and y components of the fluid velocity $\mathbf{v}$.

For the calculations we perform in this paper, we use periodic boundary conditions in the y-direction and a front-like, Dirichlet conditions in the x-direction, that is,

$$T_{1,j} = 1, \quad T_{N_x,j} = 0, \quad \text{for } j = 1, 2, ..., N_y \qquad (8)$$

These boundary conditions are easily incorporated into the above nonlinear system.

The numerical techniques we use to solve the G-equation are based on the work of Zhu and Ronney (1994) and we skip the discussion here.

3 Determination of the Flame Front: The Sliced Moment Technique

To estimate the flame propagation speed it is necessary to define the position of the flame front for a reaction layer with a finite thickness. This may be naively obtained through the tracking of a particular temperature contour. For example, we can pick a temperature T_* between 0 and 1, then use the level set $T = T_*$ to define the flame front. However, fronts defined this way usually involve some numerical smoothing and may introduce some oscillation when the numerical flame speed is calculated by finite difference approximation. The definition we use here is based on an idea used for an infinitely thin planar flame propagating in one direction extended to the case of a wrinkled flame front possessing a finite thickness. We refer to this method described below as the sliced moment technique. Consider the following one-dimensional case: At time t, the temperature distribution experiences a jump at the front $s(t)$ so the fluid is converted from the unburnt state to the burnt state and the unburnt (burnt) region is given by $x > s(t)$ ($x < s(t)$), respectively. Assuming that the temperature is a constant in both regions, the temperature profile can be expressed in the following form using the Heaviside function:

$$T(x) = T_b + H(x - s(t))(T_u - T_b), \tag{9}$$

where T_u and T_b are the unburnt and burnt temperatures respectively. The front position $s(t)$ in this case can be interpreted as

$$s(t) = \int_{-\infty}^{\infty} x\delta(x - s(t))dx = \frac{1}{T_u - T_b}\int_{-\infty}^{\infty} xT'(x)dx, \tag{10}$$

by using the definition of the Heaviside and delta functions in the sense of distribution. Obviously, this definition of the front position in terms of the moment of the temperature field extends naturally to planar flame fronts involving a finite transition layer. For wrinkled flame fronts, this definition can be extended only under some conditions. Basically, we need the slice of the temperature profile for a fixed y value to be monotone. Fronts that fold onto themselves will not satisfy this condition. Assuming that the flame fronts considered here satisfy this monotonicity requirement, we define the flame front position, $s(y, t)$, for each fixed value of y by

$$s(y, t) = \frac{1}{T_u - T_b}\int_{-\infty}^{\infty} x\frac{\partial T}{\partial x}dx. \tag{11}$$

By connecting these positions for different values of y, we will have a two-dimensional curve representing the two-dimensional front. To calculate the flame speed, we average the expression given in Eq. (11) over the y variable, $\tilde{s} = \frac{1}{L_y}\int_0^{L_y} s(y,t)dy$. We then define the average flame speed, S, by

$$S = \frac{d\tilde{s}}{dt}. \tag{12}$$

This definition of the front is quite reasonable provided the front does not fold onto itself. We additionally comment that the sliced moment technique works quite well for the flow geometries considered in this paper.

In this work, we are interested in the enhanced front propagation phenomenon resulted from a given velocity field. For this reason, we define the enhanced flame speed for a flame with thickness ϵ in a flow field with flow intensity λ

$$S_E = S(\lambda, \epsilon) - S(0, \epsilon) \tag{13}$$

in the following discussions.

4 Results

Our numerical domain is taken to be $\{(x, y) : 0 \leq x \leq 8, 0 \leq y \leq 2\}$. We assume that the wave is coming from the left and the initial front is set at $x = 2$. Specifically, the boundary condition is set so $T = 1$ at the left end of the domain and $T = 0$ at the right end of the domain, and T is assumed to be periodic in y with $T(x, y + 2, t) = T(x, y, t)$. We take the molecular diffusion coefficient, $\bar{\kappa} = 1.$, and pick the nonlinear reaction by

$$f(T) \quad = \quad T(1-T) \ .$$

This insures that the minimum speed traveling wave in the absence of a fluid flow will move with $speed = 2$.

4.1 Pure Time Fluctuation

We first document the results of simulations performed on the following purely time-dependent flow field, $\mathbf{v} = (u, v)$:

$$\begin{aligned} u &= -2 + \sin(\epsilon^{-\alpha} t) \\ v &= 0 \end{aligned}$$

and we take $\alpha = 0.99$. We make this selection of α to demonstrate the capability of the numerical scheme to resolve highly rapid temporal fluctuations, while demonstrating convergence to the predictions of Majda and Souganidis (1994). Further, the mean flow is chosen to remove the natural propagation of the pure reaction-diffusion system. Each simulation for this flow field is performed with a 300×75 spatial mesh. Figure 1 depicts the time dependent enhanced flame speed, S_E, for the case of $\epsilon = 0.05$. Observe, that there is a startup period in time due to our particular choice of the initial temperature profile, then the flame speed becomes periodic, and oscillates around the zero speed.

Note to printer: please place Figure 1 near this place in the text.

In Figure 2, we show the enhanced flame speed for the reduced value of $\epsilon = 0.02$. Observe that the frequency of oscillations has increased, whereas the amplitude is roughly the same as the previous case. There is a slight shift of the temporal mean in this case. We observe that in higher resolution studies with increased spatial resolution, this shift decreases to zero.

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Also, we remark that in all of our simulations, the computed front speed decreases with increased spatial resolution, suggesting that the calculations predict enhanced flame speeds slightly in excess of the exact value. In a forthcoming paper, we will demonstrate that this generic overprediction in the computed effective front speed is a direct consequence of the implicit numerical scheme utilized here (McLaughlin and Zhu, 1996). This observation is important in that it allows for the possibility of simple error extrapolation by elimination of the complexity associated with non-monotone convergence, at least for a pure reaction-diffusion system in the absence of a fluid flow. We will employ this observation in section 4.3 below to demonstrate the finite ϵ effect in the context of a shear layer in the presence of a transverse mean wind. Returning to the two simulations considered in this section, we clearly see weak convergence to the asymptotic predictions of Majda and Souganidis (1994) who predict in this case that the effective front propagates without seeing the turbulent flow and does not lead to any enhancement of the propagation speed. Of course, the front location, the time integral of the front speed, converges strongly to the asymptotic predictions.

4.2 Steady Shear Layer

We next consider the case of a shear layer in the absence of a transverse mean component. In each of these simulations, we use a spatial mesh consisting of 600×150 grid points. The flow we consider will be the following

$$\begin{aligned} u(x,y) &= -2 + \lambda \sin(ky) \\ v(x,y) &= 0 \ . \end{aligned}$$

Observe, that the flow field here differs from that considered by Majda and Souganidis in that the frequency, k, of the shear layer is not tied to the parameter, ϵ. We will pick $\epsilon = 0.02$ and $k = 4\pi$ for all calculations involving this flow geometry. We emphasize that with this selection of parameters, we obtain an integral scale set by by the box size, $L_y = 2.$, an intermediate turbulence scale set by $L_y/2k = 0.318$, and the size of a reaction zone $\epsilon/L_x = 0.0025$. This selection precisely captures the separation of scales considered by the asymptotic theory of Majda and Souganidis (1994) for a case involving a finite, non-zero reaction zone. We observe that front shape quickly locks into a profile set by the shear layer and initial front profile. Below we document two cases corresponding to two different initial front profiles exhibiting that the limiting profile demonstrates memory of the initial front shape. In Figure 3 we exhibit a typical series of time snapshots corresponding to $t = 0$, $t = 1.$, and $t = 2.$ depicting the dynamic approach to this limiting profile for the case of an initial pure planar front. Here, the value of the turbulent intensity parameter is $\lambda = 2$. Figure 4 depicts a similar series of time snapshots for the same value of the turbulent intensity, but for an initial sinusoidal front profile which is out of phase with the turbulent shear layer. Observe, that by the time, $t = 2.$, the front has converged to its limiting profile and further this limiting front shape is quite different than the previous case involving the initial planar front.

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Note to printer: please place Figure 4 near this place in the text.

Figure 5 depicts the time dependence of the effective front speed for this simulation. We compute this effective front speed utilizing the sliced moment technique described above in Section 3 with the average over the y variable computed using 8 uniformly distributed sample points. We remark that the approach to the limiting front speed depicted in this figure is qualitatively the same as all the simulations without a transverse mean flow described below.

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We now present the first piece of convincing evidence for the success of the Majda-Souganidis averaging theory as well as the original predictions of Damkohler (1940), in predicting the dependence of the effective flame speed on the turbulent intensity even for cases involving a finite front thickness. Figure 6 exhibits the dependence of the averaged enhanced front speed as a function of the turbulent intensity, λ. Each data point was obtained through simulations run to time $t = 2.$ for an initial planar front with varying values of λ. Importantly, we comment that in each simulation, the averaged front speed converges to a constant in a similar fashion to that depicted in Figure 5 by the time $t = 2.0$. We use these limiting values as the data to construct Figure 6. Observe that this data is co-linear, in agreement with the theoretical predictions. Exact comparison of the computed speed and the asymptotic predictions is difficult due to numerical discretization error, however the qualitative similarity is striking. We will address the issue of exact comparison in the next section for cases involving a transverse mean wind.

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4.3 Time Varying Shear Layer: The Effect of a Transverse Mean Wind

In the previous section we considered the case of a large scale mean wind which was precisely aligned with the direction of effective front propagation and whose magnitude was selected to remove the natural laminar propagation speed to contain the front within the computational domain. In such a situation, the mean flow has no effect on the enhanced flame speed since such a mean flow may be trivially removed via a Galilean transformation. However, this case is considerably different when considering transverse mean winds in a shear layer geometry for the system is no longer Galilean invariant. This lack of Galilean invariance leads to an important effect on the effective flame velocity. Embid, Majda and Souganidis have shown by nonlinear averaging the reaction-diffusion-advection system in the limit as the parameter ϵ approaches zero that the transverse mean wind may in fact reduce the enhanced flame speed. This sensitivity in the enhanced flame speed is further complicated through additional dependence on the angle between the imposed mean wind and the shearing direction. As this angle is varied, the enhanced burning speed may exhibit a region of quadratic scaling in the turbulent intensity, as opposed to the linear scaling demonstrated in the previous section in the absence of a transverse mean wind. The interested reader should consult the paper of Embid, Majda, and Souganidis (1995a) for full details.

Here, we investigate this issue of sensitive dependence on the transverse mean wind in the context of a flame front possessing a finite reaction zone. First we note that for a constant mean flow, $\mathbf{V} = (U, V)$, the Galilean transformation

$$\begin{aligned} x' &= x - Ut \\ y' &= y - Vt \end{aligned}$$

will eliminate this mean flow in the temperature equation in these new variables. However, the time-independent small-scale velocity field will now become time-dependent signifying the lack of Galilean invariance in this case. In the new variables, the full fluid flow field will be

$$\begin{aligned} u(x', y', t) &= -2 + \lambda \sin(k(y' + Vt)) \\ v(x', y', t) &= 0. \end{aligned}$$

Unless otherwise noted, each of the simulations described below utilize a spatial mesh of 600×150 grid points, and calculations are run till $t = 8$. In Figure 7, we plot the flame fronts at certain times for a representative value of the turbulent intensity, $\lambda = 0.5$ and for two different values of the transverse mean velocity, 0.5 and 1.5, for which the theory of Embid, Majda, and Souganidis guarantees both quadratic and linear regions of scaling as the turbulent intensity is varied. Within each frame, the initial front and two fronts at later times $t = 4$ and 8 are plotted. The ϵ value is chosen to be 0.02 for these two cases. It is obvious that the large transverse mean flow (the case $V = 1.5$) actually enhances the front propagation at a lower rate, compared to the other case ($V = 0.5$), in total agreement with the rigorous asymptotic theory derived for infinitely thin flame fronts.

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We see that the mean flow can have very different effects on the flame propagation, depending on its direction with respect to the direction of the flame propagation. In Figure 8, we show two plots depicting the dependence of the enhanced flame speed on the turbulent intensity computed from the full system in Eq. (1) and simulations of the G-equation given in Eq. (3). In the simulations of the G-equation, a constant laminar burning velocity $S_L = 2$ is chosen to be able to compare with that of the reation-diffusion-advection system in Eq.(1). The top plot corresponds to simulations performed with the transverse mean component, $V = .5$, whereas the lower plot corresponds to $V = 1.5$. The data for this plot was obtained in identical fashion as the method utilized in the previous section for the construction of Figure 6. Two different values of ϵ (0.05 and 0.02) are used for the full simulation of equation Eq. (1), and the finite ϵ effect can be demonstrated by direct comparison.

Note to printer: please place Figure 8 near this place in the text.

Firstly, we describe the extrapolation technique employed to obtain the data for Figure 8. Here, we study the enhanced speed of propagation *beyond* the natural laminar speed arising in the absence of a fluid flow, and refer to this quantity as the enhanced flame speed. The difficulty in comparing the computed enhanced flame speed for different values of ϵ is that the discretization error associated with any numerical scheme is intimately tied to the value of ϵ. Below we describe a reasonable method to obtain an accurate comparison.

Recall that in the previous section we noted that the implicit numerical scheme utilized in this paper always overpredicts the laminar propagation speed as a direct result of numerical error (McLaughlin and Zhu, 1996). By direct consequence, we may conclude that the error in the effective front speed, for fixed spatial meshes, increases with decreasing values of ϵ due to the lower resolution of the thinning reaction layer. To avoid this complication which leads to extreme difficulty in comparing simulations obtained for different values of ϵ and λ, we calculate S_E for each ϵ according to the difference $S(\lambda, \epsilon) - S(0, \epsilon)$. Theoretically, $S(0, \epsilon)$ is the traveling wave speed of the front in the absence of any fluid flows and it is independent of ϵ. However, in finite difference approximations it exhibits a ϵ dependence which is caused by the discretization error. In table 1, the discrepencies between the theoretical speed 2, and the numerically predicted values of the speed for different values of ϵ are given. Since we are calculating the numerical enhanced flame speed, we use the numerical approximation for $S(0, \epsilon)$ in the following calculations of S_E.

ϵ	0.05	0.02
$S(0, \epsilon)$	0.00491966	0.05861462

Table 1. Numerical errors for the planar traveling wave speed.

We remark that higher resolution calculations performed on a spatial mesh of 1200×300 indeed yields a reduction in the computed flame speed $S(\lambda, \epsilon)$ which is approximately the same for all values of λ, strongly justifying the removal of a constant $S(0, \epsilon)$ for each curve data depicted in Figure 8. Moreover, the overprediction of the velocity of the minimum speed traveling wave in the discrete solution to Eq. (1) obtained using the numerical scheme in Eq. (4) suggests that the computed flame speed will monotonically converge to the exact value, further justifying the extrapolation utilized here. Ultimate justification of this extrapolation technique will come with the advent of faster and larger computers and more elaborate numerical analysis.

Note to printer: please place Table I near this place in the text.

Observe that the top graph in Figure 8, corresponding to simulations performed with a small value of the transverse mean component, $V = .5$, does indeed demonstrate a quadratic bending for very small values of the turbulent intensity and further reverts to linear scaling at moderate values of λ in agreement with asymptotic predictions by Embid, Majda and Souganidis (1995a) in the context of infinitely thin flame fronts. Additionally, we clearly see the effect of a finite reaction zone. Namely, the enhanced flame speed for fixed values of the turbulent intensity in fact increases with decreasing values of ϵ. Moreover, the simulations of the full system in Eq. (1) underpredict enhanced flame speeds obtained from simulation of the G-equation. Further, the lower graph in Figure 8, corresponding to simulations performed with $V = 1.5$, also supports these observations. In this case, the region of quadratic scaling is seen to be much larger than the previous case, demonstrating another subtle point of the rigorous averaging theory which predicts an increased range of quadratic scaling in the turbulent intensity as the magnitude of the transverse mean wind is increased.

It is very important to understand the physical mechanism which produces this complex dependence of the enhanced flame speed on the transverse mean flow. Here we offer the following explanation of this phenomena in terms of an important interaction between front wrinkling and the magnitude of the transverse mean wind. The addition of a mean wind orthogonal to the propagation direction of a wrinkled front yields a shifting of this front in directions orthogonal to the direction of original propagation. This transverse shifting may be interpreted as effectively smoothing the wrinkled front, creating an effective layer which is thicker than the original reaction zone in the absence of the transverse mean wind. This smoothing mechanism suggests that the effective speed of propagation results from an interaction between the relative magnitude of transverse shifting and the enhanced propagation speed resulting from front wrinkling associated with the small-scale fluid flow. Consequently, the more transverse shifting we have, the flatter this effective layer becomes, and the thicker it becomes. In light of the observation made in this paper that thicker reaction zones lead to reduced effective speeds of propagation, the effect of increasing the transverse mean wind is to reduce the enhanced flame speed.

Another interesting feature demonstrated by these numerical simulations is that a thin flame is more susceptible to the local flow fluctuation than thick flames. This is clear from comparison of the enhanced flame speeds depicted in Figure 8 for different ϵ values for both strong and weak transverse mean winds. The curves corresponding to smaller reaction zones certainly admit larger variation with respect to the turbulent intensity parameter, λ, than the curves corresponding to larger reaction zones.

Finally, when we compare the results between the full nonlinear system and the G-equation, it should be pointed out that the curve for the G-equation lies above those of the nonlinear system, at least for these two ϵ values. It is noted that the flame stretch effect is not considered in our G-equation simulations. This is based on the consideration that the flame stretch effect will diminish with the flame thickness ϵ approaches zero. For the nonlinear system, as we have observed, the basic trend as ϵ is decreased is for the enhanced flame speed to increase and consequently, the curves shown in Figure 8 are seen to approach that of the G-equation. Of course it is impossible, at this stage, to compute the $\epsilon \searrow 0$ limit of either the full nonlinear system or the G-equation numerically. However, presumably nonlinear homogenized averaging will produce these limits. We remark that the work of Embid, Majda, and Souganidis (1995b) computes this limit utilizing such nonlinear averaging for similar flow geometries as considered in this paper. In that work it is observed that the effective speed of propagation of the full nonlinear system always exceeds that predicted by the the nonlinear averaging of the G-equation. We comment that further numerical studies suggest that the G-equation simulations presented here have nearly converged to the homogenized limit. Consequently, the observation made here that the enhanced flame speed predicted by the G-equation exceeds the flame speed computed from the full system in fact demontrates an important additional finite reaction zone effect. We emphasize that this phenomenon is in no way an artifact of discretization error. In fact, disrectization error leads to overprediction of computed effective flame speeds in the full nonlinear system. This surprising observation suggests a potential critical reaction zone volume below which the enhanced speed of propagation in the full system agrees with the theoretical predictions of Embid, Majda, and Souganidis (1995b) and exceeds predictions of the G-equation. We additionally comment that for the flow configurations considered here, this nonlinear averaging theory in fact agrees quite well with the nonlinear averaging of the G-equation with the discrepancy between these two predictions being quite small; however, this discrepancy has been shown to be quite large when considering effective propagation in directions different than the shearing direction (Embid, Majda, Souganidis 1995b). Our numerical studies here are entirely restricted to considering effective propagation rates along the channel in the direction of the shear flow.

Overall, we can see that the asymptotic nonlinear averaging procedure does provide an excellent approximation for the small ϵ nonlinear reaction-diffusion-advection system. We also note that the similar quadratic dependence of the enhanced flame speed on the turbulent intensity was derived from the Clavin-Williams theory (1975) for small turbulent intensities. However, the quadratic behavior observed hear is caused by a transverse mean flow and a simple small scale shear flow, while the behavior in Clavin-Williams theory is generated in the presence of isotropic turbulent flows in the absence of a large scale mean. It is interesting to see the qualitative similarities between these two very different flow geometries.

5 Conclusion

We have seen through high resolution numerical simulation an excellent agreement between the enhanced flame speeds as predicted by the theory developed by Majda and Souganidis (1994) for the context of an infinitely thin flame front, and actual flame speeds measured computationally for fronts possessing a finite, non-zero front thickness. We remark that the interesting subtle dependence of the enhanced flame speed on the transverse mean flow within the context of a turbulent shear layer is not an artifact of the nonlinear averaging theory, but is in fact visible in real simulations involving chemical reactions occurring within a finite, positive volume. Additionally, through numerical simulations and careful error extrapolation, we have shown that the presence of a finite reaction layer leads to reduced propagation speeds below those predicted by the G-equation. Further, this computed enhanced burning speed is seen to be monotonically increasing as the volume of the reaction zone is decreased, suggesting that this speed is below the speeds predicted by rigorous homogenized averaging. It is an extremely interesting question to consider if these sorts of subtle effects arise in systems possessing chemistry other than KPP type. We hope to explore these issues in the future.

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FIGURE CAPTIONS

1. The flame speed in a purely time-dependent uniform flow as a function of time, for $\epsilon = 0.05$.

2. The flame speed in a purely time-dependent uniform flow as a function of time, for $\epsilon = 0.02$.

3. Time snapshots of the flame fronts at $t = 0, 1$ and 2, for a planar initial front, with $\lambda = 2$.

4. Time snapshots of the flame fronts at $t = 0, 1$ and 2, for a sinusoidal initial front, with $\lambda = 2$.

5. The flame speed as a function of time, in a typical shear flow, for a sinusoidal front, with $\lambda = 2$.

6. The enhanced flame speed as a function of λ in a shear flow.

7. Time snapshots of the flame fronts at $t = 0, 4$ and 8, for a planar initial front, in a shear flow with $\lambda = 0.5$, $\epsilon = 0.02$. (a), with a weak transverse mean $V = 0.5$; (b), with a strong transverse mean wind $V = 1.5$.

8. The enhanced flame speeds calculated from the G-equation and the fully nonlinear reaction-diffusion-advection system with $\epsilon = 0.05$ and 0.02, as functions of λ for the (a) weak and (b) strong transverse mean wind.