A Bibliography of Publications on the Numerical Calculation of \( \pi \)

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\[(\sin \alpha)/\alpha\] [128]. 0 [248]. 1 [261]. 1/\( \pi \) [221, 222, 315, 286]. 1/\( \pi^2 \) [240, 255, 222]. 10,000 [57]. 10,000,000 [155]. 16 [230]. 2,000 [39]. 2,576,980,370,000 [249].  
\$24.95 [219]. 2H2 [259]. b [207]. C, d [303]. e, f [218, 113, 107, 65, 38, 126, 32, 39, 40, 247, 13, 63]. e\(^{(\pi/2)}\) = \(i^i\) [15]. \( \gamma \) [77]. GL(\(n, Z\)) [110]. N [129, 163, 96, 110, 154]. \( \phi \)[220, 227]. \( \pi \) [156, 218, 273, 111, 157, 201, 266, 35, 182, 140, 112, 113, 267, 310, 28, 23, 198. 70, 78, 139, 163, 17, 107, 306, 164, 92, 95, 101, 102, 119, 258, 44, 65, 209, 18, 220, 227, 295, 228, 71, 259, 213, 88, 165, 55, 152, 214, 66, 38, 210, 37, 24, 133, 296, 4, 269, 26, 21, 290, 128, 5, 9, 10, 282, 179, 229, 143, 148, 230, 115, 185, 116, 243, 122, 126, 244, 186, 93, 117, 283, 166, 180, 72, 27, 134, 181, 22, 317, 129, 105, 135, 32, 39, 84, 231, 68, 97, 47, 29, 194]. \( \pi \) [167, 205, 57, 48, 232, 7, 211, 149, 14, 200, 40, 76, 19, 6, 58, 77, 271, 69, 11, 1, 36, 247, 174, 249, 307, 94, 62, 123, 30, 175, 217, 131, 16, 13, 145, 168, 299, 155, 53, 190, 63, 8, 169]. \( \pi, e \) [87, 106]. \( \pi/12 \) [31]. \( \pi/4 \) [46]. \( \pi/8 \) [31]. \( \pi = 2 \sum \text{arccot} f_{2k+1} \) [79]. \( \pi^2 \) [257, 276, 125, 48]. \( \pi^4 \) [104]. \( \pi \text{coth} \pi \) [233]. \( q \) [243]. \( \sqrt{2} \) [61, 64]. \( \sqrt{2 + \sqrt{2}} \) [247]. \( \sum 1/k^2 = \pi^2/6 \) [67]. \( \sum_{k=1}^{\infty} 1/k^2 = \pi^2/6 \) [54]. \( \sum_{k=1}^{\infty} = \pi^2/6 \) [73]. \( \sum_{n=1}^{\infty} 1/n^2 = \pi^2/6 \) [108]. \( \sqrt{2} \) [87]. Z [110]. \( \zeta(2) = \pi^2/6 \) [285].
BBP-Type, Bechmann, Benford, Berggren, Berkeley, Best, Better, Between, Beyond, Bilateral, Billion, Billionth, Binary, Biography, Birth, Biruni, Bit, Bodleian, Book, Both, Bouncing, Boy, Brent, Bresenham, Brief, Brothers, Brouncker, Brun.

C, Calculate, Calculated, Calculates, Calculation, Calculations, Callaghan, Carnegie, Carnegie-Mellon, Catalan, Catalan-Type, Catalan’s, Catalogue, Celebrating, Celebration, Central, Century, Certain, Challenge, Changes, Chaos, Character, Charles, Chi, Chisquared, Chie, Chie, China, Chinese, Chinoise, Chongzhi, Choong, Christmas, Chronology, Cifre, Circle, Circulaires, Circular, Claims, Class, Classroom, Cluster, Colin, Collapse, Collected, Collector, Comments, Communicating, Comp, Comparative, Compendium, Complex, Complexity, Comprising, Computation, Computations, Compute, Computers, Computing, Concerning, Conclusion, Conjecture, Conjectured, Considerations, Constands, Constant, Constants, Construction, Continued, Constructors, Contributions, Convenient, Convergence, Convergent, Converging, Correct, Correspondence, Counting, Coupon, Crucible, Cruncher, Cubic, CUDA.
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External [124]. Extraction [237, 238]. Extremal [213].


REFERENCES

Systems [316, 264].


Xeon [307].

y-cruncher [313]. Year [150]. yields [129]. Youqin [168].


References

REFERENCES


[8] Carl Louis Ferdinand von Lindemann. Über die Zahl π. (German) [On the number π]. *Mathematische Annalen*, 20(??):213–225, ???? 1882. CODEN MAANA3. ISSN 0025-5831 (print), 1432-1807 (electronic). In this famous paper, von Lindemann proved that π is transcendental, showing that it is impossible to square the circle by compass and straightedge, a problem dating back before 400 BCE in Greece.

REFERENCES


REFERENCES


REFERENCES


REFERENCES


REFERENCES


[58] Daniel Shanks and John W. Wrench, Jr. Calculation of $\pi$ to 100,000 decimals. *Mathematics of Computation*, 16(77):76–99, January 1962. CODEN MCMPAF. ISSN 0025-5718 (print), 1088-6842 (electronic). URL http://www.jstor.org/stable/2003813. A note added in proof says: “J. M. Gerard of IBM United Kingdom Limited, who was then unaware of the computation described above, computed $\pi$ to 20,000 D on the 7090 in the London Data Centre on July 31, 1961. His program used Machin’s formula, (1) $[\pi = 16 \arctan(1/5) - 4 \arctan(1/239)]$, and required 39 minutes running time. His result agrees with ours to that number of decimals.”.


[64] I. J. Good and T. N. Gover. The generalized serial test and the binary expansion of \( \sqrt{2} \). *Journal of the Royal Statistical Society. Series A (General)*, 131(??):434, ???? 1968. CODEN JSSAEF. ISSN 0035-9238. See [61].


REFERENCES


REFERENCES


REFERENCES


REFERENCES

theoretical computer science of the Gesellschaft für Informatik (G.I.) and
the special interest group for applied mathematic[s] of the Association
française des sciences et techniques de l’information, de l’organisation et
des systèmes (AFCET)".

Montgomery:1985:MMT

[99] Peter L. Montgomery. Modular multiplication without trial division.
MCMPAF. ISSN 0025-5718 (print), 1088-6842 (electronic). URL http://

Newman:1985:SVF

[100] D. J. Newman. A simplified version of the fast algorithms of Brent and
CODEN MCMPAF. ISSN 0025-5718 (print), 1088-6842 (electronic).

Borwein:1986:ECI

26 (1):123–126, March 1986. CODEN BITTEL, NBITAB. ISSN 0006-3835
(print), 1572-9125 (electronic). URL http://www.springerlink.com/
openurl.asp?genre=article&issn=0006-3835&volume=26&issue=1&spage=
123.

Borwein:1986:MQC

[102] J. M. Borwein and P. B. Borwein. More quadratically converging al-
1986. CODEN MCMPAF. ISSN 0025-5718 (print), 1088-6842 (electronic).

Ferguson:1986:SPE

[103] H. R. P. Ferguson. A short proof of the existence of vector Euclidean
algorithms. Proceedings of the American Mathematical Society, 97(??):
8–10, ?? 1986. CODEN PAMYAR. ISSN 0002-9939 (print), 1088-6826
(electronic). URL http://www.ams.org/mathscinet-getitem?mr=87k:
11080.

Hancl:1986:NSP

ISSN 0002-9890 (print), 1930-0972 (electronic).

Matiyasevich:1986:NNF

[105] Yuri V. Matiyasevich. Notes: A new formula for π. American Mathemat-
cal Monthly, 93(8):631–635, October 1986. CODEN AMMYAE. ISSN
0002-9890 (print), 1930-0972 (electronic).
Parks:1986: NOI


Bernstein:1987:NFA


Choe:1987: NEP

[108] Boo Rim Choe. Notes: An elementary proof of \(\sum_{n=1}^{\infty} \frac{1}{n^2} = \pi^2/6\). *American Mathematical Monthly*, 94(7):662–663, August/September 1987. CODEN AMMYAE. ISSN 0002-9890 (print), 1930-0972 (electronic).

Edgar:1987: PDE


Ferguson:1987: NIA


Almkvist:1988: GLR


Bailey:1988: CDD

REFERENCES


[19] J. M. Borwein, P. B. Borwein, and D. H. Bailey. Ramanujan, modular equations, and approximations to $\pi$ or how to compute one billion digits of
REFERENCES


REFERENCES


REFERENCES


Mauron:1992:P


Abeles:1993:CDG


Arno:1993:NPT


Bailey:1993:AMT


Beckmann:1993:HP


Badger:1994:LLA

REFERENCES


Adamchik:1997:NSF

[156] Victor Adamchik and Stan Wagon. Notes: A simple formula for $\pi$. *American Mathematical Monthly*, 104(9):852–855, November 1997. CODEN AMMYAE. ISSN 0002-9890 (print), 1930-0972 (electronic). URL http://www.maa.org/pubs/monthly_nov97_toc.html. The authors employ Mathematica to extend earlier work of Bailey, Borwein [119], and Plouffe, [159], done in 1995, but only just published, that discovered an amazing formula for $\pi$ as is a power series in $16^{-k}$, enabling any base-16 digit of $\pi$ to be computed without knowledge of any prior digits. In this paper, Mathematica is used to find several simpler formulas having powers of $4^{-k}$. They also note that it has been proven that their methods cannot be used to exhibit similar formulas in powers of $10^{-k}$.

Almkvist:1997:MCD


Bailey:1997:QP


Bailey:1997:RCV


Bailey:1997:RME


Bailey:1997:RNC


Bellard:1997:BBD


Bellard:1997:NFC

[163] Fabrice Bellard. A new formula to compute the n-th binary digit of \pi. This formula is claimed in [248] to be somewhat faster to compute than the BBP formula., 1997. URL http://bellard.org/pi/pi_bin.pdf.

Blatner:1997:JP


Delahaye:1997:FNc


Laczkovich:1997:LPI


Ogawa:1997:BEC


Volkov:1997:ZYH


REFERENCES


REFERENCES


REFERENCES


REFERENCES


REFERENCES


REFERENCES


REFERENCES


[236] United States Congress. House Resolution 224: Pi day. Web document, March 12, 2009. The resolution ends with: “Resolved, That the House of Representatives—(1) supports the designation of a “Pi Day” and its celebration around the world; (2) recognizes the continuing importance of National Science Foundation’s math and science education programs; and (3) encourages schools and educators to observe the day with appropriate activities that teach students about Pi and engage them about the study of mathematics.”.


[257] David H. Bailey, Jonathan M. Borwein, Andrew Mattingly, and Glenn Wightwick. The computation of previously inaccessible digits of \( \pi^2 \) and Catalan’s constant. Report, Lawrence Berkeley National Laboratory; Centre for Computer Assisted Research Mathematics and its Applications (CARMA), University of Newcastle; IBM Australia, Berkeley, CA, USA; Callaghan, NSW 2308, Australia; St. Leonards, NSW 2065, Australia; Pyrmont, NSW 2009, Australia, April 11, 2011. 18 pp. URL http://crd.lbl.gov/~dhbailey/dhbpapers/bbp-bluegene.pdf.

[258] D. Borwein and Jonathan M. Borwein. Proof of some experimentally conjectured formulas for \( \pi \). Preprint, Department of Mathematics, University of Western Ontario and Centre for Computer-assisted Research Mathematics and its Applications (CARMA), School of Mathematical and Physical Sciences, University of Newcastle, London, ON, Canada and Callaghan, NSW 2308, Australia, December 4, 2011.


REFERENCES


REFERENCES


Füks:2012:AAK


Osada:2012:EHC


Shelburne:2012:ED


Agarwal:2013:BGC


Alladi:2013:R


AragonArtacho:2013:WRN

REFERENCES


REFERENCES


[287] Alexander Yee and Shiguro Kondo. It stands at 10 trillion digits of \( \pi \)... world record for both desktop and supercomputer!!! Web site, April 15, 2013. URL http://www.numberworld.org/y-cruncher/. This site also contains a table of digit records from 2009 to 2013 for various mathematical constants. The \( \pi \) record of 10,000,000,000,050 decimal digits was reached on 17 October 2011 after 371 days of computation, and 45 hours of verification.
REFERENCES


[290] Reinhard E. Ganz. The decimal expansion of π is not statistically random. *Experimental mathematics*, 23(2):99–104, 2014. CODEN ????? ISSN 1058-6458 (print), 1944-950X (electronic). See the reproduction of results, and reanalysis, in [300], that reveals a flaw in the statistical analysis in this paper: Ganz used only a single blocksize in sampling digits, and that blocksize produces anomalous statistics.


REFERENCES


REFERENCES


