A Bibliography of Publications on the Numerical Calculation of \( \pi \)

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\[(\sin \alpha) / \alpha \text{[127]}, 0 \text{[245]}, 1 \text{[258]}, 1/\pi \text{[280, 219, 220]}, 1/\pi^2 \text{[252, 220]}, 10,000 \text{[57]}, 10,000,000 \text{[154]}, 16 \text{[228]}, 2 \text{[60, 63]}, 2 + 2 \text{[244]}, 2,000 \text{[39]}, 2,576,980,370,000 \text{[246]}, \textbf{24,95} \text{[217]}, 29,360,000 \text{[111]}, zH_2 \text{[256]}, b \text{[205]}, C \text{[297]}, d \text{[297]}, e \text{[216, 112, 106, 64, 38, 125, 32, 39, 40, 244, 13, 62]}, e^{-\pi/2} = i^1 \text{[15]}, e \text{[86, 105]}, \gamma \text{[76]}, GL(n, Z) \text{[109]}, N \text{[128, 162, 95, 109, 153]}, \phi \text{[218, 225]}, \pi \text{[270, 139, 264, 138, 300, 164, 118, 207, 289, 70, 87, 212, 290, 284, 277, 133, 178, 128, 96, 230, 209, 14, 76, 301, 173, 167, 293, 155, 216, 110, 156, 198, 263, 35, 111, 112, 28, 23, 195, 69, 77, 162, 17, 106, 91, 94, 100, 101, 255, 44, 64, 18, 218, 225, 226, 256, 211, 55, 151, 65, 38, 208, 37, 24, 132, 4, 266, 26, 21, 127, 5, 9, 10, 176, 227, 142, 147, 228, 114, 182, 115, 240, 121, 125, 241, 183, 92, 116, 165, 177, 71, 27, 22, 104, 134, 32, 39, 83, 229]}, \pi \text{[67, 47, 29, 191, 166, 202, 57, 48, 7, 148, 197, 40, 75, 19, 6, 58, 268, 68, 11, 12, 36, 244, 172, 246, 93, 61, 122, 30, 215, 130, 16, 13, 144, 154, 53, 187, 62, 8]}, \pi, e \text{[86, 105]}, \pi/12 \text{[31]}, \pi/4 \text{[46]}, \pi/8 \text{[31]}, \pi = 2 \sum \arccot f_{2k+1} \text{[78]}, \pi^2 \text{[254, 124, 48]}, \pi^4 \text{[103]}, \pi \coth \pi \text{[231]}, q \text{[240]}, \sum 1/k^2 = \pi^2/6 \text{[66]}, \sum_{k=1}^\infty 1/k^2 = \pi^2/6 \text{[54]}, \sum_{k=1}^\infty \pi^2/6 \text{[72]}, \sum_{n=1}^\infty 1/n^2 = \pi^2/6 \text{[107]}, \sqrt{2} \text{[86]}, Z \text{[109]}, \zeta(2) = \pi^2/6 \text{[279]}. \]

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London Data Centre on July 31, 1961. His program used Machin’s formula, (1) \[ \pi = 16 \arctan(1/5) - 4 \arctan(1/239) \], and required 39 minutes running time. His result agrees with ours to that number of decimals.”.


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(3) encourages schools and educators to observe the day with appropriate activities that teach students about Pi and engage them about the study of mathematics.”.

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