A Bibliography of Publications on the Numerical Calculation of \( \pi \)

Nelson H. F. Beebe
University of Utah
Department of Mathematics, 110 LCB
155 S 1400 E RM 233
Salt Lake City, UT 84112-0090
USA

Tel: +1 801 581 5254
FAX: +1 801 581 4148
E-mail: beebe@math.utah.edu, beebe@acm.org, beebe@computer.org (Internet)
WWW URL: http://www.math.utah.edu/~beebe/

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\[(\sin \alpha) / \alpha = 0.0 \] [248]. 1 [261]. 1/\pi [221, 222, 315, 286]. 1/\pi^2 [240, 255, 222]. 10,000 [57]. 10,000,000 [155]. 16 [230]. 2,000 [39]. 2,576,980,370,000 [249].

$24.95 [219]. \sqrt{\pi} [259]. b [207]. C [303]. d [303]. e [218, 113, 107, 65, 38, 126, 32, 39, 40, 247, 13, 63]. e^{-(\pi/2)} = i [15]. \gamma [77].

GL(n, Z) [110]. N [129, 163, 96, 110, 154]. \phi [220, 227]. \pi [156, 218, 273, 111, 157, 201, 266, 35, 182, 140, 112, 113, 267, 310, 28, 23, 198, 70, 78, 139, 163, 17, 107, 306, 164, 92, 95, 101, 102, 119, 258, 44, 65, 209, 18, 220, 227, 295, 228, 71, 259, 213, 88, 320, 165, 55, 152, 214, 66, 38, 210, 37, 24, 133, 296, 4, 269, 26, 21, 290, 128, 5, 9, 10, 282, 179, 229, 143, 148, 230, 115, 185, 116, 243, 122, 126, 244, 186, 93, 117, 283, 166, 180, 72, 27, 134, 181, 22, 317, 129, 105, 135, 32, 39, 84, 231, 68, 97, 47]. \pi [29, 194, 167, 205, 57, 48, 232, 7, 211, 149, 14, 200, 40, 76, 19, 6, 58, 77, 271, 69, 11, 12, 36, 247, 174, 249, 307, 94, 62, 123, 30, 175, 217, 131, 16, 13, 145, 168, 299, 155, 53, 190, 63, 8, 169]. \pi, e [87, 106]. \pi/12 [31]. \pi/4 [46]. \pi/8 [31]. \pi = 2 \sum \arccot f_{2k+1} [79]. \pi^2 [257, 276, 125, 48]. \pi^4 [104]. \pi \cot \pi [233]. q [243]. \sqrt{2} [61, 64]. \sqrt{2 + \sqrt{2}} [247]. \sum 1/k^2 = \pi^2/6 [67]. \sum_{k=1}^{\infty} 1/k^2 = \pi^2/6 [54]. \sum_{k=1}^{\infty} = \pi^2/6 [73]. \sum_{n=1}^{\infty} 1/n^2 = \pi^2/6 [108]. \sqrt{2} [87]. Z [110]. \zeta(2) = \pi^2/6 [285].

0 [219]. 0-88385-900-9 [219].

1975 [322]. 1983 [323].


3rd [324].

524 [80].

719 [138]. 786 [172].

'88 [325].

9 [219]. 90 [146]. 90d [160]. 949 [299].


Approximation [35, 140, 23, 37, 26, 36, 175, 53, 186, 91, 323].


Arccotangent [27]. Archimedes [308, 289, 292]. Arctan [181].

Arctangent [17, 93, 136]. arising [97]. Arithmetic [111, 212, 80, 82, 86, 76, 172, 117, 249]. Arithmetic-Geometric [111, 76].


Aryabhata [21]. Aspects [324]. Association [219]. asymptotic [205].


Australia [333].


base [50]. base-dependence [50]. Based [146, 316, 93, 94, 280]. Bases [254, 199]. BBP
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Garage [284]. Garrity [219]. Gauss [111, 93, 134, 94]. Gave [301]. General
[250]. Generalization [83]. generalized [85, 61, 64]. generating [286].
geometriae [133]. Geometric [111, 76, 81, 136]. German [8]. goodness

Hadoop [248]. Happy [291]. harmonic [129]. held [323, 322]. Helped
[292]. Hennecke [30]. Heterogeneous [316]. hexadecimal [307].
Hierarchical [124]. High [212, 268, 264]. High-Precision [212, 268].
Higher [83, 95]. histoire [116]. Historical [15, 11, 12]. History
[70, 78, 294, 142, 139, 116, 270]. Hold [196]. Honnecourt [123]. Honor
[331]. House [236]. Hui [145]. hydrogen [297]. hyperbolic [9, 10].
Hypergeometric [286, 264].

ibid [77]. Identically [185]. Identities [170]. if [278, 288]. implementation
[279, 280, 249]. Inaccessible [257, 276]. incomputability [242].
Independent [185]. Indian [25]. Inductive [110]. Infinite [66, 247, 7].
Institute [323]. Integer [183, 127, 192, 176, 98, 121, 144]. integral [114].
Introduction [235]. inverse [131]. inverse-tangent [131]. Involving
[113, 16, 310, 114]. Irrational [244, 106, 126]. Irrationality
[226, 72]. iteration [101]. iterations [213].

journeys [329]. Joy [164].

Kočański [269]. Kreis [30].

Lazzarini [140]. lecture [260]. Legacy [333]. Legendre [93, 94]. Leibnitz
[46]. Leibnitz-Gregory [46]. Leibniz [321]. less [223]. level [315]. levels
[299]. Lucas [143]. Lucky [140].

M [333]. Machin [207, 227, 200, 131]. Machin-Type [207, 227]. Magical
[292]. manuscript [133, 230, 123]. Many [157]. Math [204, 297, 77].
Mathematical


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Xeon [307].

y-crvuncher [313, 318]. Year [150]. yields [129]. Youqin [168].


References

Lambert:1768:MQP


Shanks:1853:CMC


Frisby:1871:C


Glaisher:1871:RC


Shanks:1873:ENV


Polster:1879:NIS


vonLindemann:1882:ZGN

[8] Carl Louis Ferdinand von Lindemann. Über die Zahl π. (German) [On the number π]. *Mathematische Annalen*, 20(??):213–225, ????, 1882. CODEN MAANA3. ISSN 0025-5831 (print), 1432-1807 (electronic). In this famous paper, von Lindemann proved that π is transcendental, showing that it is impossible to square the circle by compass and straightedge, a problem dating back before 400 BCE in Greece.


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[58] Daniel Shanks and John W. Wrench, Jr. Calculation of $\pi$ to 100,000 decimals. *Mathematics of Computation*, 16(77):76–99, January 1962. CODEN MCMPAF. ISSN 0025-5718 (print), 1088-6842 (electronic). URL http://www.jstor.org/stable/2003813. A note added in proof says: “J. M. Gerard of IBM United Kingdom Limited, who was then unaware of the computation described above, computed $\pi$ to 20,000 D on the 7090 in the London Data Centre on July 31, 1961. His program used Machin’s formula, (1) $\pi = 16 \arctan(1/5) - 4 \arctan(1/239)$, and required 39 minutes running time. His result agrees with ours to that number of decimals.”.


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Lauro:1972:SDS


Papadimitriou:1973:CNS


Brent:1976:FMP


Brent:1976:MPZ


Salamin:1976:CUA


Shanks:1976:TER

Mathematics of Computation, 30(134):381, 1976. CODEN MCMPAF.

Beckmann:1977:HP


Anderson:1978:F


Brent:1978:AMF


Solomon:1978:GP


Brent:1979:RMF


Ferguson:1979:GEA


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Ferguson:1986:SPE


Hancl:1986:NSP

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Mauron:1992:P


Abeles:1993:CDG


Arno:1993:NPT


Bailey:1993:AMT


Beckmann:1993:HP


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[155] Gong Yi Wei, Zi Giang Yang, Jia Chang Sun, and Jia Kai Li. The computation of $\pi$ to 10,000,000 decimal digits. *Journal on Numerical Methods and Computer Applications*, 17(1):78–81, 1996. ISSN 1000-3266. Also

[156] Victor Adamchik and Stan Wagon. Notes: A simple formula for \(\pi\). American Mathematical Monthly, 104(9):852–855, November 1997. CODEN AMMYAE. ISSN 0002-9890 (print), 1930-0972 (electronic). URL http://www.maa.org/pubs/monthly_nov97_toc.html. The authors employ Mathematica to extend earlier work of Bailey, Borwein [119], and Plouffe, [159], done in 1995, but only just published, that discovered an amazing formula for \(\pi\) as a power series in \(16^{-k}\), enabling any base-16 digit of \(\pi\) to be computed without knowledge of any prior digits. In this paper, Mathematica is used to find several simpler formulas having powers of \(4^{-k}\). They also note that it has been proven that their methods cannot be used to exhibit similar formulas in powers of \(10^{-k}\).


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national Conference on Cloud Computing Technology and Science (Cloud-
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