A Bibliography of Publications on the Numerical Calculation of \( \pi \)

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\begin{align*}
(\pi) & [249]. \ (\sin \alpha)/\alpha [131]. \ 0 [252]. \ 1/\pi [224, 225, 319, 290]. \ 1/\pi^2 [243, 259, 225]. \ 10,000 [57]. \ 10,000,000 [158]. \ 16 [233]. \ 2,000 [39]. \ 2,576,980,370,000 [253]. \ \$24.95 [222]. \ \sqrt{x^2} [263]. \ 6 [210]. \ C [307]. \ d [307]. \ c \ [221, 116, 110, 65, 38, 129, 32, 39, 40, 251, 13, 63]. \ e^{-\pi/2} = i [15]. \ \gamma [79]. \ GL(n, Z) [113]. \ N [132, 166, 98, 113, 157]. \ \phi [223, 230]. \ \pi [320, 327, 159, 221, 277, 114, 160, 204, 270, 35, 185, 143, 115, 116, 271, 314, 28, 23, 201, 71, 80, 142, 166, 17, 110, 310, 10, 104, 105, 2, 62, 44, 65, 212, 18, 223, 230, 299, 231, 72, 263, 216, 90, 325, 168, 55, 155, 217, 67, 38, 213, 37, 75, 24, 146, 151, 233, 118, 188, 119, 246, 125, 129, 247, 189, 95, 120, 287, 169, 183, 73, 27, 137, 184, 22, 322, 13, 108, 138, 32, 39, 86, 234, 69]. \ \pi [99, 47, 29, 197, 170, 208, 57, 48, 235, 7, 214, 152, 14, 203, 40, 78, 19, 6, 58, 79, 275, 70, 11, 12, 36, 251, 177, 253, 311, 96, 62, 126, 30, 178, 220, 134, 16, 13, 148, 171, 303, 158, 53, 193, 63, 8, 172]. \ \pi, e [89, 109]. \ \pi/12 [31]. \ \pi/4 [46]. \ \pi/8 [31]. \ \pi = 2 \sum \arccot f_{2k+1} [81]. \ \pi^2 [261, 280, 128, 48]. \ \pi^4 [107]. \ \pi \coth \pi [236]. \ \pi [246]. \ \sqrt{2} [61, 64]. \ \sqrt{2} + \sqrt{2} [251]. \ \sum 1/k^2 = \pi^2/6 [68]. \ \sum k=1 1/k^2 = \pi^2/6 [54]. \ \sum k=1 \infty 1/k^2 = \pi^2/6 [74]. \ \sum n=1 \infty 1/n^2 = \pi^2/6 [111]. \ \sqrt{2} [89]. \ Z [113]. \ \zeta(2) = \pi^2/6]
\end{align*}
\]
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Borweins [115]. both [291]. Bouncing [316]. Boy [305]. Brent [88, 103].
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Conclusion [37]. Conjecture [65, 50]. Conjectured [224, 270, 262]. considerations [112].
Correspondence [62]. Counting [135, 154]. Coupon [45]. crucible [238].
cruncher [317, 323]. Cubic [97, 104]. CUDA [287].
decimales [60]. Decimals [310, 3, 58, 96, 60, 233]. Degree
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313, 152, 177, 253, 158, 291]. dimensional [132]. Dimensions [85]. Dirac
[28, 17, 18, 31, 24, 26, 19, 16]. Distant [310]. Distributed
[188, 252, 191, 177]. distribution [73]. distribuzione [73]. divided [307].

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[98, 104]. exploration [299]. Exploring [145]. Exponential [65, 246].
Expressing [230]. Expressions [65]. Extended [53]. Extension [6].

Factorial [146]. famous [301]. Farm [305]. fascinating [168]. fascinating
[168]. Fast [76, 110, 93, 103]. fastest [319]. Ferguson [87]. FFTs [127].
FGHC [151]. Fibonacci [136, 146]. fifteenth [136]. fifteenth-century
[136]. Figures [34]. Finding [173, 210, 140, 77, 244, 117, 179, 101, 124]. First
Formula [320, 159, 81, 28, 325, 108, 54, 234, 74, 68, 309, 166, 300, 301].
Formule [210, 184, 263]. Formulas [240, 241, 242, 254, 255, 256, 257, 258,
204, 260, 324, 237, 230, 259, 270, 279, 314, 262, 189]. FORTRAN
[141, 149, 82, 84]. Fortran-90 [149]. Found [325]. Fractals [133]. Fraction
French [168, 60, 119, 2]. Function [327, 77, 244, 246]. Functions
Fundamental [196, 121]. further [75].


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Zach [303]. Zahl [30, 8]. Zero [257, 265, 77, 244]. zero-finding [77, 244]. Zhao [171].
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MAANA3. ISSN 0025-5831 (print), 1432-1807 (electronic). In this famous paper, von Lindemann proved that $\pi$ is transcendental, showing that it is impossible to square the circle by compass and straightedge, a problem dating back before 400 BCE in Greece.


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Based on Interim Report ADA014059, Department of Computer Science, Carnegie-Mellon University (July 1975), ii + 26 pages. See also [78] and update in [244].


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