A Bibliography of Publications on the Numerical Calculation of $\pi$

Nelson H. F. Beebe  
University of Utah  
Department of Mathematics, 110 LCB  
155 S 1400 E RM 233  
Salt Lake City, UT 84112-0090  
USA  
Tel: +1 801 581 5254  
FAX: +1 801 581 4148  
E-mail: beebe@math.utah.edu, beebe@acm.org, beebe@computer.org (Internet)  
WWW URL: http://www.math.utah.edu/~beebe/

30 April 2019  
Version 1.94

($\sin \alpha$)/$\alpha$ [127]. 0 [245]. 1 [258]. 1/$\pi$ [280, 219, 220]. 1/$\pi^2$ [252, 220]. 10,000 [57]. 10,000,000 [154]. 16 [228]. 2 [60, 63]. 2 + 2 [244]. 2,000 [39]. 2,576,980,370,000 [246]. $\mathbf{824.95}$ [217]. 29,360,000 [111]. $2 H_2$ [256]. $b$ [205]. $C$ [297]. $d$ [297]. $e$ [216, 112, 106, 38, 32, 40, 244, 13, 62]. $e^{-(\pi/2)} = i^2$ [15]. $\gamma$ [76]. $GL(n, Z)$ [109]. $N$ [128, 162, 95, 109, 153]. $\phi$ [218, 225]. $\pi$ [270, 139, 264, 138, 300, 164, 118, 207, 289, 70, 87, 212, 290, 284, 277, 133, 178, 128, 96, 230, 209, 14, 76, 301, 173, 167, 293, 155, 216, 110, 156, 198, 263, 35, 111, 112, 28, 23, 195, 69, 77, 162, 17, 106, 91, 94, 100, 101, 255, 44, 64, 18, 218, 225, 226, 256, 211, 55, 151, 65, 38, 208, 37, 24, 132, 4, 266, 26, 21, 127, 5, 9, 10, 176, 227, 142, 147, 228, 114, 182, 115, 240, 121, 125, 241, 183, 92, 116, 165, 177, 71, 27, 22, 104, 134, 32, 39, 83, 229]. $\pi$ [67, 47, 29, 191, 166, 202, 57, 48, 7, 148, 197, 40, 75, 19, 6, 58, 268, 68, 11, 12, 36, 244, 172, 246, 93, 61, 122, 30, 215, 130, 14, 131, 154, 53, 187, 62, 8]. $\pi, e$ [86, 105]. $\pi/12$ [31]. $\pi/4$ [46]. $\pi/8$ [31]. $\pi = 2 \sum \arccot f_{2k+1}$ [78]. $\pi^2$ [254, 124, 48]. $\pi^4$ [103]. $\pi \coth \pi$ [231]. $q$ [240]. $\sum 1/k^2 = \pi^2/6$ [66]. $\sum_{k=1}^{\infty} 1/k^2 = \pi^2/6$ [54]. $\sum_{k=1}^{\infty} \pi^2/6$ [72]. $\sum_{n=1}^{\infty} 1/n^2 = \pi^2/6$ [107]. $\sqrt{2}$ [86]. $Z$ [109]. $\zeta(2) = \frac{\pi^2}{6}$ [279].

0 [217]. 0-88385-900-9 [217].

1975 [303]. 1983 [304].


3rd [305].

524 [79].

719 [137]. 786 [170].

'88 [306].

9 [217]. 90 [145]. 90d [159]. 949 [293].

[302]. Adam [266]. Adamandy [266]. Adams [217]. Addenda [287].
[1]. Al [228, 19]. Al-Biruni [19]. Al-Kāshī [228]. Alexandria [309].
Algebraic [203, 55, 95, 242]. Algorithm [111, 82, 109, 146, 266, 92, 83, 93,
136, 117, 126, 274, 84, 113, 174, 88, 133, 148, 197, 143, 137, 79, 81, 170].
[121]. America [217]. American [288]. Among [57, 136, 97, 120]. Analysis
[174]. Analytic [303, 303]. ancient [68, 144, 309]. Annual [305].
applications [189]. Approach [249, 264, 135]. approximate [122].
Approximation [35, 139, 23, 37, 26, 36, 173, 53, 183, 90, 304].
Approximations [70, 266, 159, 95, 118, 14, 76]. April [303]. Arbitrary
[251]. Arccotangent [27]. Archimedes [283, 286]. Arctan [178].
Arctangent [17, 92, 135]. arising [96]. Arithmetic
[110, 210, 79, 81, 85, 75, 170, 116, 246]. Arithmetic-Geometric [110, 75].
Arithmetical [184]. Articles [288]. ary [205]. Aryabhata [21]. Aspects
AUGMENT [85]. August [304].

base-dependence [50]. Based [145, 92, 93, 275]. Bases [251, 196]. BBP
[235, 236, 237, 247, 248, 249, 250, 251, 253, 205, 258]. BBP-Type
beginnings [166]. Being [131, 150]. Bellard [299]. Benford [262].


dimensional [128]. Dimensions [82]. Dirac [240]. Direction [149].
Discovering [241]. Discovery [249, 199]. Discussions
[28, 17, 18, 31, 24, 26, 19, 16]. Distant [300]. Distributed
[182, 245, 185, 172]. distribution [71]. distribuzione [71]. divided [297].
Elementary [73, 107, 54, 74, 238]. Ellipses [110]. Empirical [264, 226].
employee [302]. energy [291]. ENIAC [283, 39, 40, 268]. enri [166].
Episodes [214]. Equally [86]. equations [159, 118, 14]. equivalent [68].
[84, 82, 89, 102]. Euler [112, 140, 202]. European [25]. Evaluation
[140, 73, 34, 74, 238]. events [212]. evidence [239]. Evolution [53]. Exact
[300]. Excluding [205]. Execution [137]. Existence [102]. Expansion
[284, 91, 60, 63]. Expansions [190, 203, 213, 48]. Experiment [206].
Experimental [140, 222, 239, 233]. Experimentally [201, 263, 255].
Experimentation [199]. Explaining [1]. explicationis [1]. Explicit
[95, 100]. exploration [289]. Exploring [141]. Exponential [64, 240].
Expressing [225]. Expressions [64]. Extended [53]. Extension [6].
External [123]. Extraction [235, 236]. Extremal [211].
[84]. FFTs [123]. FGHC [147]. Fibonacci [132, 142]. fifteenth [132].
fifteenth-century [132]. Figures [34]. Finding
[168, 205, 136, 74, 238, 113, 174, 97, 120]. First [57, 39, 297]. Floating [210].
Floating-Point [210]. Florida [306]. Forcade [84]. forgotten [243]. Form
[65]. Formal [248, 300]. Formula
[155, 78, 28, 104, 54, 229, 72, 66, 299, 162, 290, 291]. Formulae
[205, 178, 256]. Formulas
FORTRAN [137, 145, 79, 81]. Fortran-90 [145]. Fractals [129]. Fraction
French [59, 115, 2]. function [74, 238, 240]. Functions
[73, 304, 170, 195, 280]. Fundamental [190, 117].
Garage [278]. Garrity [217]. Gauss [110, 92, 133, 93]. Gave [295]. General
[247]. Generalization [82]. generalized [84, 60, 63]. generating [280].
generation [274]. Generator [152, 275]. Generators [194, 212].
geometriae [132]. Geometric [110, 75, 80, 135]. German [8]. Google [302].
Grand [141]. graphics [274]. Great [217, 216]. greco [224]. Greek


R [76]. Rabbinical [173]. Ramanujan
[270, 308, 110, 252, 159, 118, 256, 20, 56, 181, 186]. Ramanujan-like


REFERENCES

units [274]. University [304, 303]. Unleashed [188]. Unless [285].

Value [21, 19, 6, 68]. Values [31, 39]. various [158, 153]. Vector [102].
Vectorization [116]. version [99]. versus [90]. very [91, 7]. via [168].
Villard [122]. vita [224].

Wales [295]. Walking [271]. Wallis [290, 229, 244]. Wallis-
[244]. Ward
[87]. Way [201]. which [216, 7]. while [291]. who [297]. Without [98].

Xeon [301].

Year [149]. yields [128]. Youqin [167].

Zach [293]. Zahl [30, 8]. Zero [250, 258, 74, 238]. zero-finding [74, 238].
Zhao [167].

References

[1] Leonhard Euler. Testamen explicationis phaenomenorum aeris. (Latin)
contents/euler/e007tr.pdf. Translation to English, and annotations,
by Ian Bruce.

des quantités transcendentes circulaires et logarithmiques. (French) [Note
on some remarkable properties of circular and logarithmic transcendental
In this famous paper, Lambert proved that π is irrational. See [165] for
further remarks, a simplification of the proof, and references to earlier
papers that discuss Lambert’s proof.

Rectification of the Circle to 607 Places of Decimals. G. Bell, London,
UK, 1853. xvi + 95 + 1 pp. LCCN QA467 .S53 1853.
REFERENCES


[8] Carl Louis Ferdinand von Lindemann. Über die Zahl \(\pi\). (German) [On the number \(\pi\)]. *Mathematische Annalen, 20(??):213–225, ???? 1882. CODEN MAANA3. ISSN 0025-5831 (print), 1432-1807 (electronic). In this famous paper, von Lindemann proved that \(\pi\) is transcendental, showing that it is impossible to square the circle by compass and straightedge, a problem dating back before 400 BCE in Greece.


REFERENCES


REFERENCES


REFERENCES


REFERENCES


[54] Yoshio Matsuoka. Mathematical notes: An elementary proof of the formula \( \sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6} \). *American Mathematical Monthly*, 68(5):485–487, May 1961. CODEN AMMYAE. ISSN 0002-9890 (print), 1930-0972 (electronic).


[58] Daniel Shanks and John W. Wrench, Jr. Calculation of \( \pi \) to 100,000 decimals. *Mathematics of Computation*, 16(77):76–99, January 1962. CODEN MCMMAF. ISSN 0025-5718 (print), 1088-6842 (electronic). URL http://www.jstor.org/stable/2003813. A note added in proof says: “J. M. Gerard of IBM United Kingdom Limited, who was then unaware of the computation described above, computed \( \pi \) to 20,000 D on the 7090 in the
London Data Centre on July 31, 1961. His program used Machin’s formula, (1) \[ \pi = 16 \arctan(1/5) - 4 \arctan(1/239) \], and required 39 minutes running time. His result agrees with ours to that number of decimals.”

ESMENJAUD-BONNARDEL:1965:ESD


GOOD:1967:GST


TEE:1967:CP


YARBROUGH:1967:PCC


GOOD:1968:GST

[63] I. J. Good and T. N. Gover. The generalized serial test and the binary expansion of \( \sqrt{2} \). *Journal of the Royal Statistical Society. Series A (General)*, 131(??):434, ????. 1968. CODEN JSSAEF. ISSN 0035-9238. See [60].

BROWN:1969:REE


DRAIM:1969:FCF

E. L. Stark. Classroom notes: Another proof of the formula $\sum \frac{1}{k^2} = \frac{\pi^2}{6}$. *American Mathematical Monthly*, 76(5):552–553, May 1969. CODEN AMMYAE. ISSN 0002-9890 (print), 1930-0972 (electronic).


Ioannis Papadimitriou. Classroom notes: a simple proof of the formula $\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$. *American Mathematical Monthly*, 80(4):424–425, April 1973. CODEN AMMYAE. ISSN 0002-9890 (print), 1930-0972 (electronic).

REFERENCES


REFERENCES


REFERENCES


REFERENCES

[94] J. M. Borwein and P. B. Borwein. Cubic and higher order algorithms for \( \pi \).
*Bulletin canadien de mathématiques = Canadian Mathematical Bulletin*,
27(??):436–443, ???. 1984. CODEN CMBUA3. ISSN 0008-4395 (print),
1496-4287 (electronic).

[95] J. M. Borwein and P. B. Borwein. Explicit algebraic \( n \)th order approxima-
tions to \( \pi \). In Singh et al. [304], pages 247–256. ISBN 94-009-6466-8, 94-
009-6468-4. ISSN 1389-2185. LCCN ???. URL http://link.springer.
com/chapter/10.1007/978-94-009-6466-2_12.

[96] Morris Newman and Daniel Shanks. On a sequence arising in series for \( \pi \).
MCMPAF. ISSN 0025-5718 (print), 1088-6842 (electronic).

[97] J. Håstad, B. Helfrich, J. Lagarias, and C. P. Schnorr. Polynomial time al-
tgorithms for finding integer relations among real numbers. In Monien and
Vidal-Naquet [305], pages 105–118. CODEN LNCSD9. ISBN 0-387-16078-
7 (paperback). ISSN 0302-9743 (print), 1611-3349 (electronic). LCCN
service/series/0558/tocs/t0210.htm;
Organized jointly by the special interest group for
theoretical computer science of the Gesellschaft für Informatik (G.I.)
and the special interest group for applied mathematic[s] of the Association
française des sciences et techniques de l’information, de l’organisation et
des systèmes (AFCET)”.

[98] Peter L. Montgomery. Modular multiplication without trial division.
MCMPAF. ISSN 0025-5718 (print), 1088-6842 (electronic). URL http:

[99] D. J. Newman. A simplified version of the fast algorithms of Brent and
CODEN MCMPAF. ISSN 0025-5718 (print), 1088-6842 (electronic).
REFERENCES


REFERENCES


REFERENCES


REFERENCES

Jochi:1989:CMA

Tee:1989:NBA

Bailey:1990:FEH

Desbrow:1990:NI

Johnson:1990:SDC

Bailey:1991:PTN

Gillman:1991:TML


[134] C. Mauron. \( \pi \) [\( \pi \)]. Mauron and Lachat, Fribourg, Switzerland, 1992. ISBN ???????? pp. LCCN ?????? Mauron computes \( \pi \) to 1,000,000 decimal digits using independent formulas of Liebniz, Machin, and Störmer.


REFERENCES


REFERENCES


[155] Victor Adamchik and Stan Wagon. Notes: A simple formula for \( \pi \). *American Mathematical Monthly*, 104(9):852–855, November 1997. CODEN AMMYAE. ISSN 0002-9890 (print), 1930-0972 (electronic). URL http://www.maa.org/pubs/monthly_nov97_toc.html. The authors employ Mathematica to extend earlier work of Bailey, Borwein [118], and Plouffe, [158], done in 1995, but only just published, that discovered an amazing formula for \( \pi \) as is a power series in \( 16^{-k} \), enabling any base-16 digit of \( \pi \) to be computed without knowledge of any prior digits. In this paper, Mathematica is used to find several simpler formulas having powers of \( 4^{-k} \). They also note that it has been proven that their methods cannot be used to exhibit similar formulas in powers of \( 10^{-k} \).


REFERENCES

[162] Fabrice Bellard. A new formula to compute the $n$-th binary digit of $\pi$. This formula is claimed in [245] to be somewhat faster to compute than the BBP formula., 1997. URL http://bellard.org/pi/pi_bin.pdf.


REFERENCES


REFERENCES


C. Percival. PiHex: a distributed effort to calculate $\pi$. The computation took two years, and used 250 CPU years, using otherwise-idle time on 1734 machines in 56 countries., 2000. URL http://oldweb.cecm.sfu.ca/projects/pihex.


REFERENCES


REFERENCES


Tu:2005:SRD


Adams:2006:GDW


Boslaugh:2006:BRG


Chan:2006:T


Guillera:2006:CCS


Guillera:2006:NMO

REFERENCES


REFERENCES


[234] United States Congress. House Resolution 224: Pi day. Web document, March 12, 2009. The resolution ends with: “Resolved, That the House of Representatives— (1) supports the designation of a “Pi Day” and its celebration around the world; (2) recognizes the continuing importance of National Science Foundation’s math and science education programs; and
(3) encourages schools and educators to observe the day with appropriate activities that teach students about Pi and engage them about the study of mathematics.”.


REFERENCES


[254] David H. Bailey, Jonathan M. Borwein, Andrew Mattingly, and Glenn Wightwick. The computation of previously inaccessible digits of $\pi^2$ and Catalan’s constant. Report, Lawrence Berkeley National Laboratory; Centre for Computer Assisted Research Mathematics and its Applications (CARMA), University of Newcastle; IBM Australia, Berkeley, CA, USA; Callaghan, NSW 2308, Australia; St. Leonards, NSW 2065, Australia; Pyrmont, NSW 2009, Australia, April 11, 2011. 18 pp. URL http://crd.lbl.gov/~dhbailey/dhbpapers/bbp-bluegene.pdf.
REFERENCES

[255] D. Borwein and Jonathan M. Borwein. Proof of some experimentally conjectured formulas for $\pi$. Preprint, Department of Mathematics, University of Western Ontario and Centre for Computer-assisted Research Mathematics and its Applications (CARMA), School of Mathematical and Physical Sciences, University of Newcastle, London, ON, Canada and Callaghan, NSW 2308, Australia, December 4, 2011.


REFERENCES


REFERENCES


[281] Alexander Yee and Shiguro Kondo. It stands at 10 trillion digits of pi... world record for both desktop and supercomputer!!! Web site, April 15, 2013. URL http://www.numberworld.org/y-cruncher/. This site also
contains a table of digit records from 2009 to 2013 for various mathematical constants. The $\pi$ record of 10,000,000,005 decimal digits was reached on 17 October 2011 after 371 days of computation, and 45 hours of verification.


[284] Reinhard E. Ganz. The decimal expansion of $\pi$ is not statistically random. *Experimental mathematics*, 23(2):99–104, 2014. CODEN ????? ISSN 1058-6458 (print), 1944-950X (electronic). See the reproduction of results, and reanalysis, in [294], that reveals a flaw in the statistical analysis in this paper: Ganz used only a single blocksize in sampling digits, and that blocksize produces anomalous statistics.


REFERENCES


[Takahashi:2018:CQH] Daisuke Takahashi. Computation of the 100 quadrillionth hexadecimal digit of \( \pi \) on a cluster of Intel Xeon Phi processors. *Parallel Comput-


