A Bibliography of Publications on the Numerical Calculation of π

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\( \sin \alpha / \alpha \) \[127\]. \( 0 \) \[240\]. \( 1 \) \[253\]. \( 1/\pi \) \[275\], \( 215 \), \( 216 \]. \( 1/\pi^2 \) \[247\], \( 216 \]. \( 10,000 \) \[57\]. \( 10,000,000 \) \[152\]. \( 16 \) \[224\]. \( 2 \) \[60\], \( 63 \]. \( 2 + 2 \) \[239\]. \( 2,000 \) \[39\]. \( 2,576,980,370,000 \) \[241\]. \( 29,360,000 \) \[111\]. \( 2H_2 \) \[251\]. \( b \) \[203\]. \( e \)
\[112\], \( 106 \), \( 64 \), \( 38 \), \( 125 \), \( 32 \), \( 39 \), \( 40 \), \( 239 \), \( 13 \), \( 62 \]. \( e^{-\pi^2/2} = i^\gamma \) \[15\]. \( \gamma \) \[76\].
\( GL(n, Z) \) \[109\]. \( N \) \[128\], \( 160 \), \( 95 \), \( 109 \), \( 151 \]. \( \phi \) \[214\], \( 221 \]. \( \pi \)
\[265\], \( 138 \), \( 259 \), \( 118 \), \( 205 \), \( 284 \), \( 70 \), \( 87 \), \( 210 \), \( 285 \), \( 279 \), \( 272 \), \( 132 \), \( 176 \), \( 128 \), \( 96 \), \( 226 \), \( 207 \), \( 14 \), \( 76 \), \( 171 \), \( 165 \), \( 288 \), \( 153 \), \( 110 \), \( 154 \), \( 196 \), \( 258 \), \( 35 \), \( 111 \), \( 112 \), \( 28 \), \( 23 \), \( 193 \), \( 69 \), \( 77 \), \( 137 \), \( 160 \), \( 17 \), \( 106 \), \( 162 \), \( 91 \), \( 94 \), \( 100 \), \( 101 \), \( 250 \), \( 44 \), \( 64 \), \( 18 \), \( 214 \), \( 221 \), \( 222 \), \( 225 \), \( 209 \), \( 55 \), \( 149 \), \( 65 \), \( 38 \), \( 206 \), \( 37 \), \( 24 \), \( 131 \), \( 4 \), \( 261 \), \( 26 \), \( 21 \), \( 127 \), \( 5 \), \( 9 \), \( 10 \), \( 174 \), \( 223 \), \( 140 \), \( 145 \), \( 224 \), \( 114 \), \( 180 \), \( 115 \), \( 235 \), \( 121 \), \( 125 \), \( 236 \), \( 181 \), \( 92 \), \( 116 \), \( 163 \), \( 175 \), \( 71 \), \( 72 \), \( 22 \), \( 104 \), \( 133 \), \( 32 \), \( 39 \), \( 83 \), \( 225 \), \( 67 \), \( 47 \), \( 29 \]. \( \pi \)
\[189\], \( 164 \), \( 200 \), \( 57 \), \( 48 \), \( 7 \), \( 146 \), \( 195 \), \( 40 \), \( 75 \), \( 19 \), \( 6 \), \( 58 \), \( 263 \), \( 68 \), \( 11 \), \( 12 \), \( 36 \), \( 239 \), \( 170 \), \( 241 \), \( 93 \), \( 162 \), \( 30 \), \( 213 \), \( 129 \), \( 16 \), \( 13 \), \( 142 \), \( 152 \), \( 53 \), \( 185 \), \( 62 \), \( 8 \]. \( \pi \), \( e \) \[86\], \( 105\]. \( \pi/12 \) \[31\]. \( \pi/4 \) \[46\]. \( \pi/8 \) \[31\]. \( \pi = 2 \sum \arccot f_{2k+1} \) \[78\]. \( \pi^2 \) \[249\], \( 124 \), \( 48 \]. \( \pi^4 \)
\[103\]. \( \pi \coth \pi \) \[227\]. \( q \) \[235\]. \( \sum 1/k^2 = \pi^2/6 \) \[66\]. \( \sum_{k=1}^\infty 1/k^2 = \pi^2/6 \) \[54\].
\( \sum_{k=1}^\infty \pi^2/6 \) \[72\]. \( \sum_{n=1}^\infty 1/n^2 = \pi^2/6 \) \[107\]. \( \sqrt{2} \) \[86\]. \( Z \) \[109\]. \( \zeta(2) = \pi^2/6 \) \[274\].

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vonLindemann:1882:ZGN

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Glaisher:1883:CHL


Glaisher:1891:CHL


Smith:1895:HSA


Smith:1896:EHS


Veblen:1904:T


Ramanujan:1914:MEA

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Dodge:1996:NRN


Plouffe:1996:CTD


[153] Victor Adamchik and Stan Wagon. Notes: A simple formula for \( \pi \). *American Mathematical Monthly*, 104(9):852–855, November 1997. CODEN AMMYAE. ISSN 0002-9890 (print), 1930-0972 (electronic). URL http://www.maa.org/pubs/monthly_nov97_toc.html. The authors employ Mathematica to extend earlier work of Bailey, Borwein [118], and Plouffe, [156], done in 1995, but only just published, that discovered an amazing formula for \( \pi \) as is a power series in \( 16^{-k} \), enabling any base-16 digit of \( \pi \) to be computed without knowledge of any prior digits. In this paper, Mathematica is used to find several simpler formulas having powers of \( 4^{-k} \). They also note that it has been proven that their methods cannot be used to exhibit similar formulas in powers of \( 10^{-k} \).


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of Representatives— (1) supports the designation of a “Pi Day” and its celebration around the world; (2) recognizes the continuing importance of National Science Foundation’s math and science education programs; and (3) encourages schools and educators to observe the day with appropriate activities that teach students about Pi and engage them about the study of mathematics.”.

Adegoke:2010:NBD

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