A Bibliography of Publications on the Numerical Calculation of $\pi$

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(sin $\alpha$)/$\alpha$ [127]. 0 [240]. 1 [253]. 1/$\pi$ [275, 215, 216]. 1/$\pi^2$ [247, 216]. 10,000 [57]. 10,000,000 [152]. 16 [224]. 2 [60, 63]. 2 + 2 [239]. 2,000 [39]. 2,576,980,370,000 [241]. 29,360,000 [111]. $2H_2$ [251]. $b$ [203]. $e$
[112, 106, 64, 38, 125, 32, 39, 40, 239, 13, 62]. $e^{-1/2}$ = $i$ [15]. $\gamma$ [76].

GL($n$, $\mathbb{Z}$) [109]. $N$ [128, 160, 95, 109, 151]. $\phi$ [214, 221]. $\pi$
[189, 164, 200, 57, 48, 7, 146, 195, 40, 75, 19, 6, 58, 263, 68, 11, 12, 36, 239, 170, 241, 93, 61, 122, 30, 213, 129, 16, 13, 142, 152, 53, 185, 62, 8]. $\pi, e$ [86, 105]. $\pi/12$ [31]. $\pi/4$ [46]. $\pi/8$ [31]. $\pi = 2 \arccot f_{2k+1}$ [78]. $\pi^2$ [249, 124, 48]. $\pi^4$
[103]. $\pi \coth \pi$ [227]. $q$ [235]. $\sum 1/k^2 = \pi^2/6$ [66]. $\sum_{k=1}^\infty 1/k^2 = \pi^2/6$ [54].

$\sum_{k=1}^\infty \pi^2/6$ [72]. $\sum_{n=1}^\infty 1/n^2 = \pi^2/6$ [107]. $\sqrt{2}$ [86]. $Z$ [109]. $\zeta(2) = \pi^2/6$ [274].

1975 [293]. 1983 [294].


3rd [295].

524 [79].

719 [136]. 786 [168].

'88 [296].

90 [143]. 90d [157]. 949 [288].

Again [277, 268]. ages [189]. Air [1]. Al [224, 19]. Al-Biruni [19].
Attempts [11, 12]. AUGMENT [85]. August [294].

beginnings [164]. being [130, 148]. Benford [257]. Berggren [299].


ibid [76]. Identically [180]. Identities [166]. if [268, 277]. implementation
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Properties [289, 211, 57, 213, 234, 38, 190].


Year [147]. yields [128]. Youqin [165].
References


[8] Carl Louis Ferdinand von Lindemann. Über die Zahl \( \pi \). (German) [On the number \( \pi \)]. *Mathematische Annalen*, 20(??):213–225, ???? 1882. CODEN MAANA3. ISSN 0025-5831 (print), 1432-1807 (electronic). In this famous paper, von Lindemann proved that \( \pi \) is transcendental, showing that it is impossible to square the circle by compass and straightedge, a problem dating back before 400 BCE in Greece.


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December 1955. CODEN AMMYAE. ISSN 0002-9890 (print), 1930-0972 (electronic).


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[60] I. J. Good and T. N. Gover. The generalized serial test and the binary expansion of $\sqrt{2}$. *Journal of the Royal Statistical Society. Series A (Gen-


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MCMPAF. ISSN 0025-5718 (print), 1088-6842 (electronic). See also [74, 233].


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Beckmann:1993:HP


Badger:1994:LLA


Bailey:1994:EEE


Hauss:1994:FLC


Rossner:1994:SIR


Volkov:1994:CAC


Bailey:1995:FBM


Finch:1995:MBB

[144] Steven Finch. The miraculous Bailey–Borwein–Plouffe π algorithm. Recent URLs redirect to an unrelated site, but the one given here worked on


Adamchik and Stan Wagon. Notes: A simple formula for π. *American Mathematical Monthly*, 104(9):852–855, November 1997. CODEN AMMYAE. ISSN 0002-9890 (print), 1930-0972 (electronic). URL http://www.maa.org/pubs/monthly_nov97_toc.html. The authors employ Mathematica to extend earlier work of Bailey, Borwein [118], and Plouffe, [156], done in 1995, but only just published, that discovered an amazing formula for π as is a power series in $16^{-k}$, enabling any base-16 digit of π to be computed without knowledge of any prior digits. In this paper, Mathematica is used to find several simpler formulas having powers of $4^{-k}$. They also note that it has been proven that their methods cannot be used to exhibit similar formulas in powers of $10^{-k}$.


D. H. Bailey, J. M. Borwein, and P. B. Borwein. Ramanujan, modular equations, and approximations to pi or How to compute one billion digits


[160] Fabrice Bellard. A new formula to compute the $n$-th binary digit of $\pi$. This formula is claimed in [240] to be somewhat faster to compute than the BBP formula., 1997. URL http://bellard.org/pi/pi_bin.pdf.


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[191] Paul Preuss. Are the digits of \( \pi \) random? A Berkeley Lab researcher may hold the key. *Energy Science News*, ??(??):??,


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of Representatives—(1) supports the designation of a “Pi Day” and its celebration around the world; (2) recognizes the continuing importance of National Science Foundation’s math and science education programs; and (3) encourages schools and educators to observe the day with appropriate activities that teach students about Pi and engage them about the study of mathematics.”.


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contains a table of digit records from 2009 to 2013 for various mathematical constants. The $\pi$ record of 10,000,000,000,050 decimal digits was reached on 17 October 2011 after 371 days of computation, and 45 hours of verification.


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**Bailey:2016:RCS**

**Roberts:2016:HFB**

**Bailey:2017:PCP**

**Yee:2017:PNL**

**Traub:1976:ACC**

**Singh:1984:ATS**

**Monien:1986:SAS**


