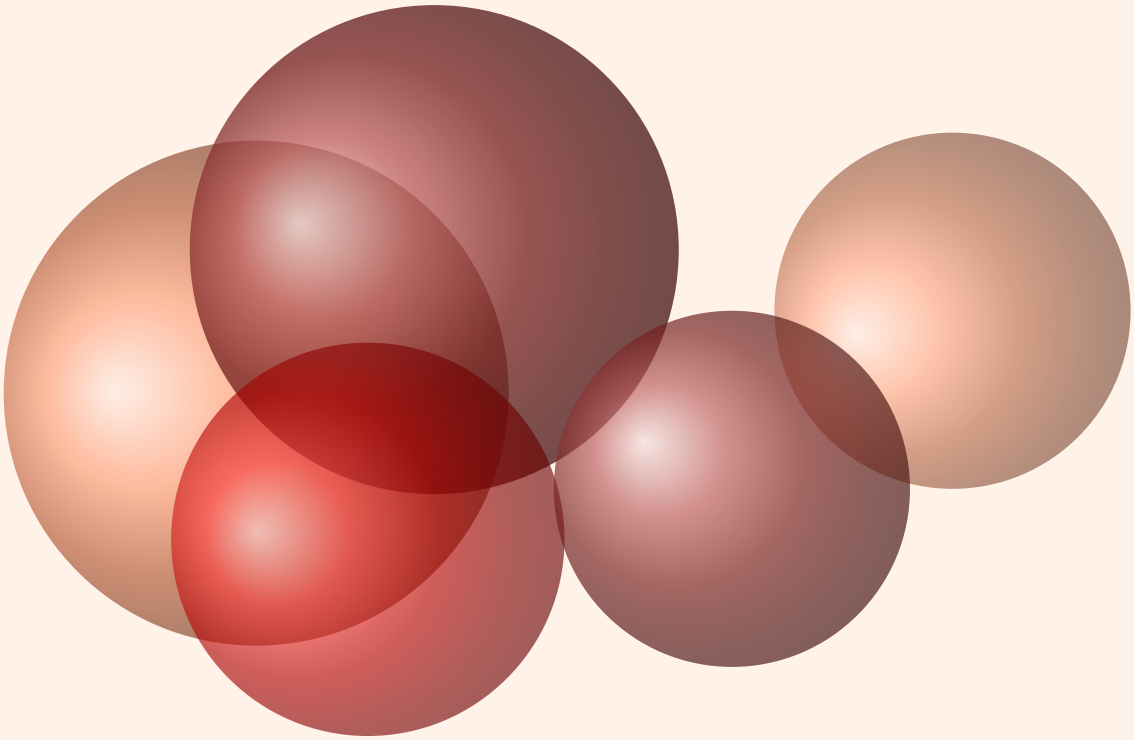
NamedGraphs v 1.00 c

AlterMundus



Alain Matthes

February 28, 2011

http://altermundus.fr http://altermundus.com

NamedGraphs

AlterMundus

Alain Matthes

NamedGraphs.pdf *is not a beginner or advanced tutorial, not a study of graphs, it's only a gallery of undirected graphs made with the package tkz-berge.sty v 1.00 c. Some of graphs have names, sometimes inspired by the graph's topology, and sometimes after their discoverer.* ***NamedGraphs.pdf*** *presents some of them. A lot of references can be found here* `http://mathworld.wolfram.com`

- Firstly, I would like to thank **Till Tantau** for the beautiful LATEX package, namely TikZ.
- I am grateful to **Michel Bovani** for providing the **`fourier`** font.
- I received much valuable advice and guidance on Graph Theory from **Rafael Villarroel** `http://graphtheoryinlatex.blogspot.com/`.
- The names of graphs can be found here MathWorld by E.Weisstein

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SECTION 1

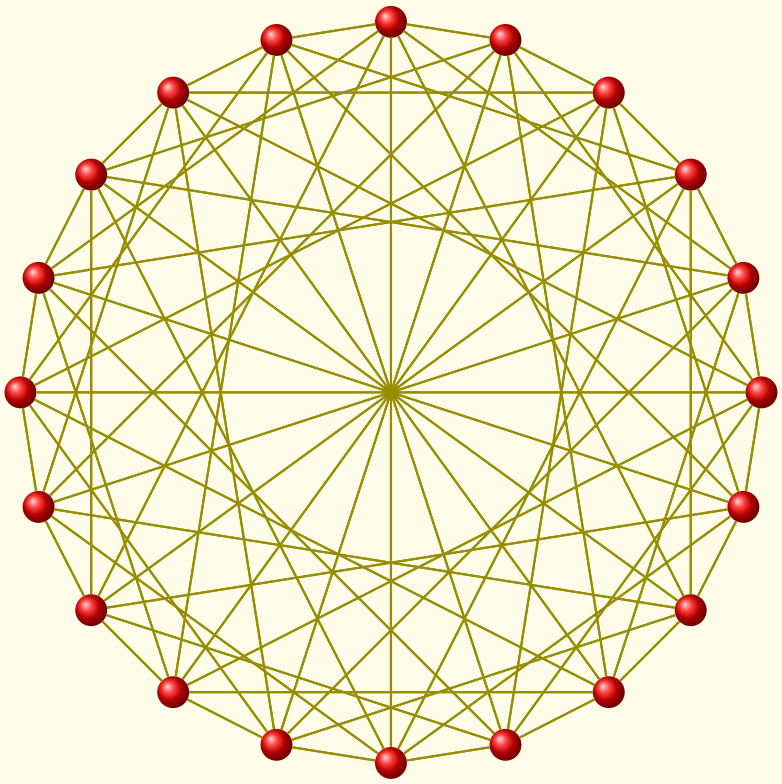
Andrasfai graph

\grAndrasfai[⟨*options*⟩]{⟨*k*⟩}

From MathWord : http://mathworld.wolfram.com/AndrasfaiGraph.html

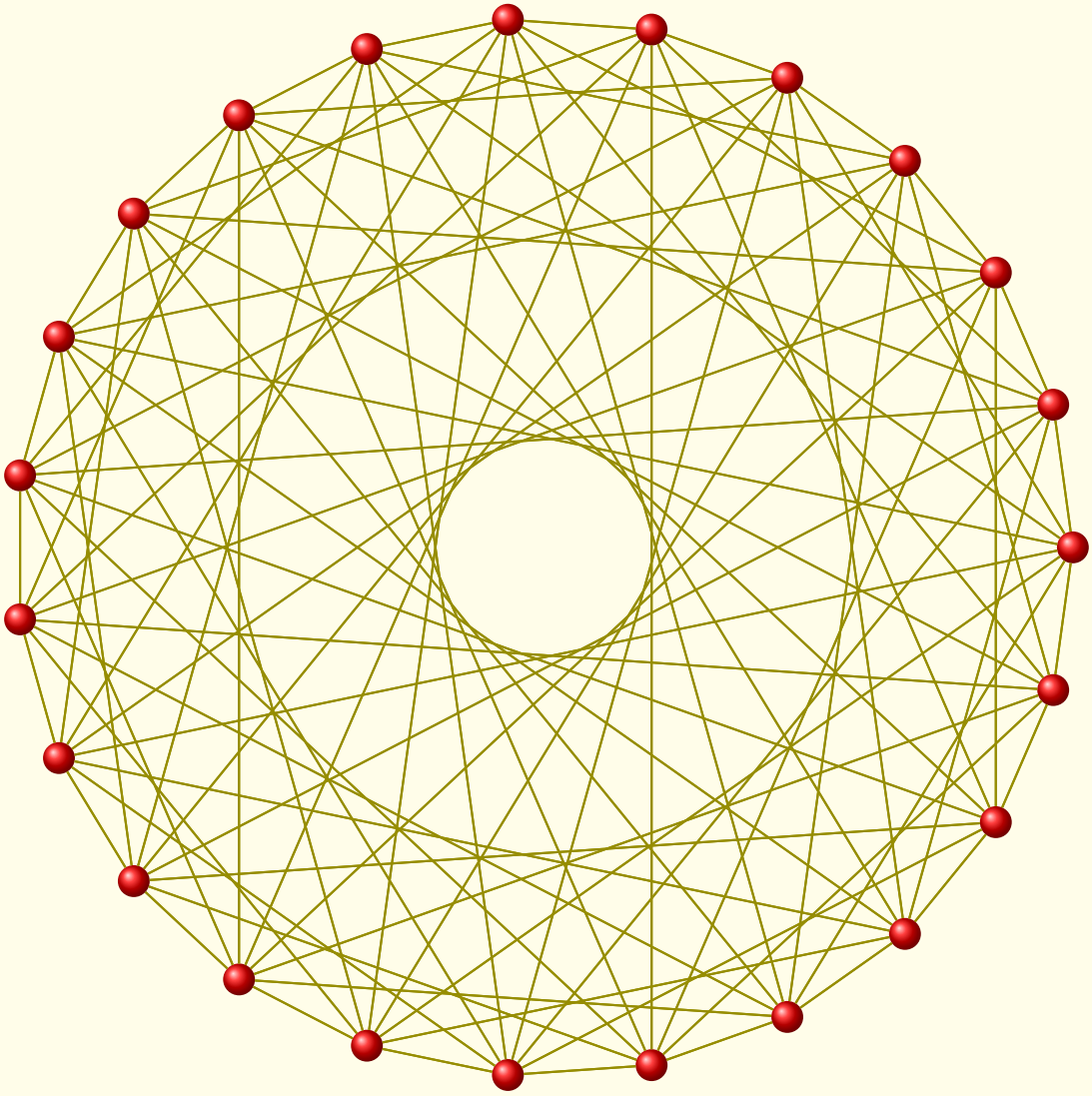
The k-Andrásfai graph is a circulant graph on $3k-1$ *nodes whose indices are given by the integers* $1,\ldots,3k-1$ *that are congruent to 1 (mod 3). MathWorld by E.Weisstein*

1.1 Andrásfai graph : k=7, order 20



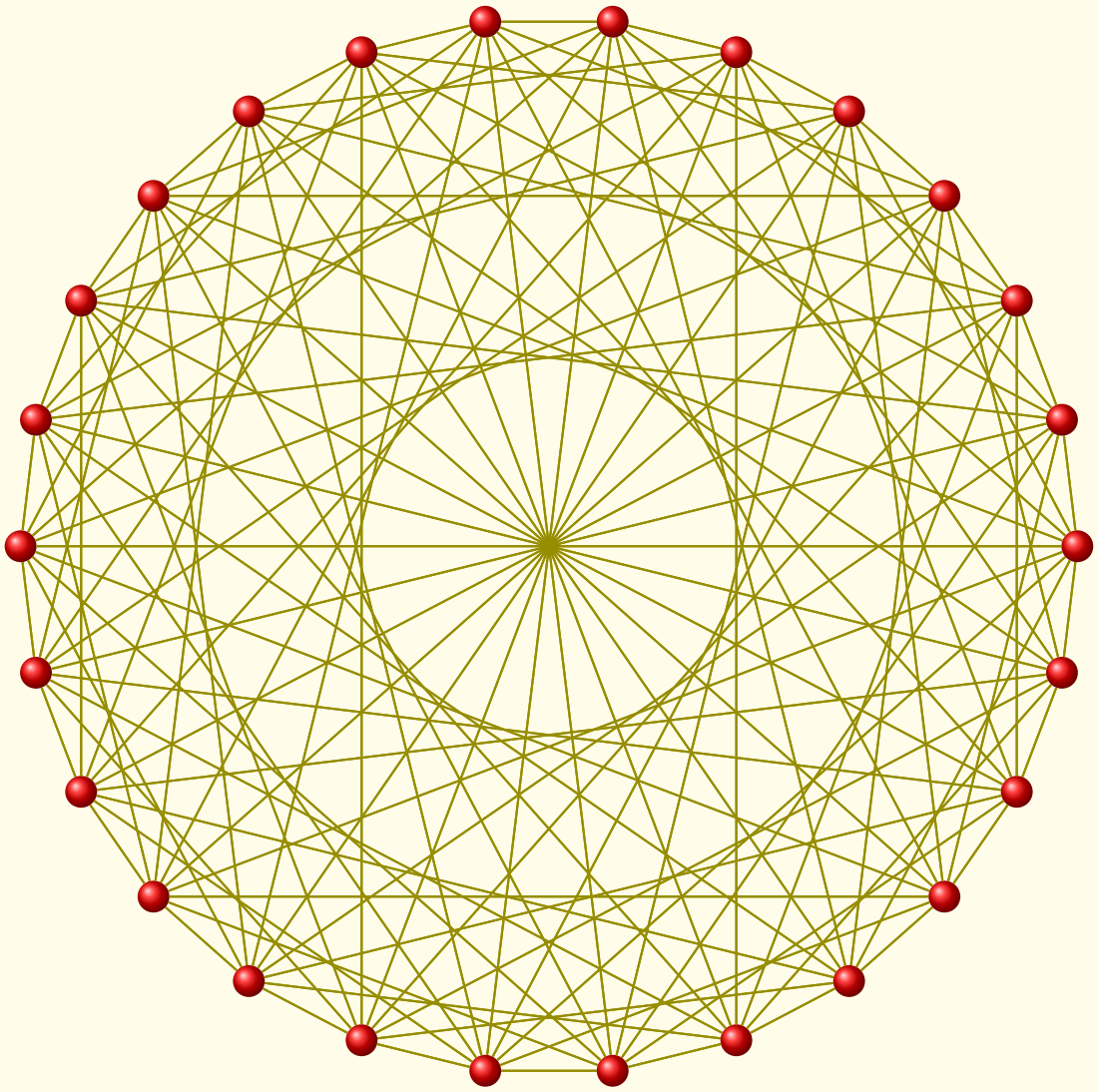
```
\begin{tikzpicture}[scale=.7]
  \GraphInit[vstyle=Art]
  \SetGraphArtColor{red}{olive}
  \grAndrasfai[RA=7]{7}
\end{tikzpicture}
```

1.2 Andrásfai graph : k=8, order 23



```
\begin{tikzpicture}
\GraphInit[vstyle=Art]
\SetGraphArtColor{red}{olive}
\grAndrasfai[RA=7]{8}
\end{tikzpicture}
```

1.3 Andrásfai graph : k=9, order 26



```
\begin{tikzpicture}
\GraphInit[vstyle=Art]
\SetGraphArtColor{red}{olive}
\grAndrasfai[RA=7]{9}
\end{tikzpicture}
```

SECTION 2

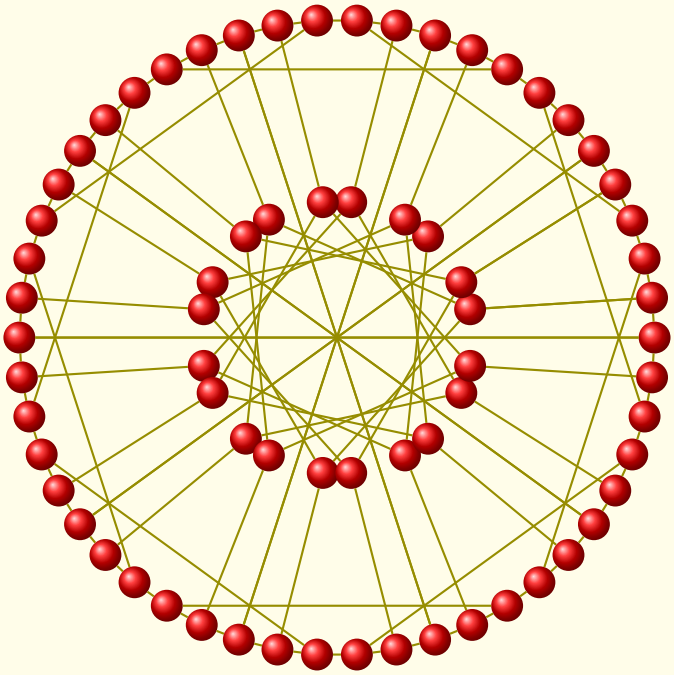
Balaban

\grBalaban[⟨*options*⟩]

From MathWord : http://mathworld.wolfram.com/Balaban10-Cage.html

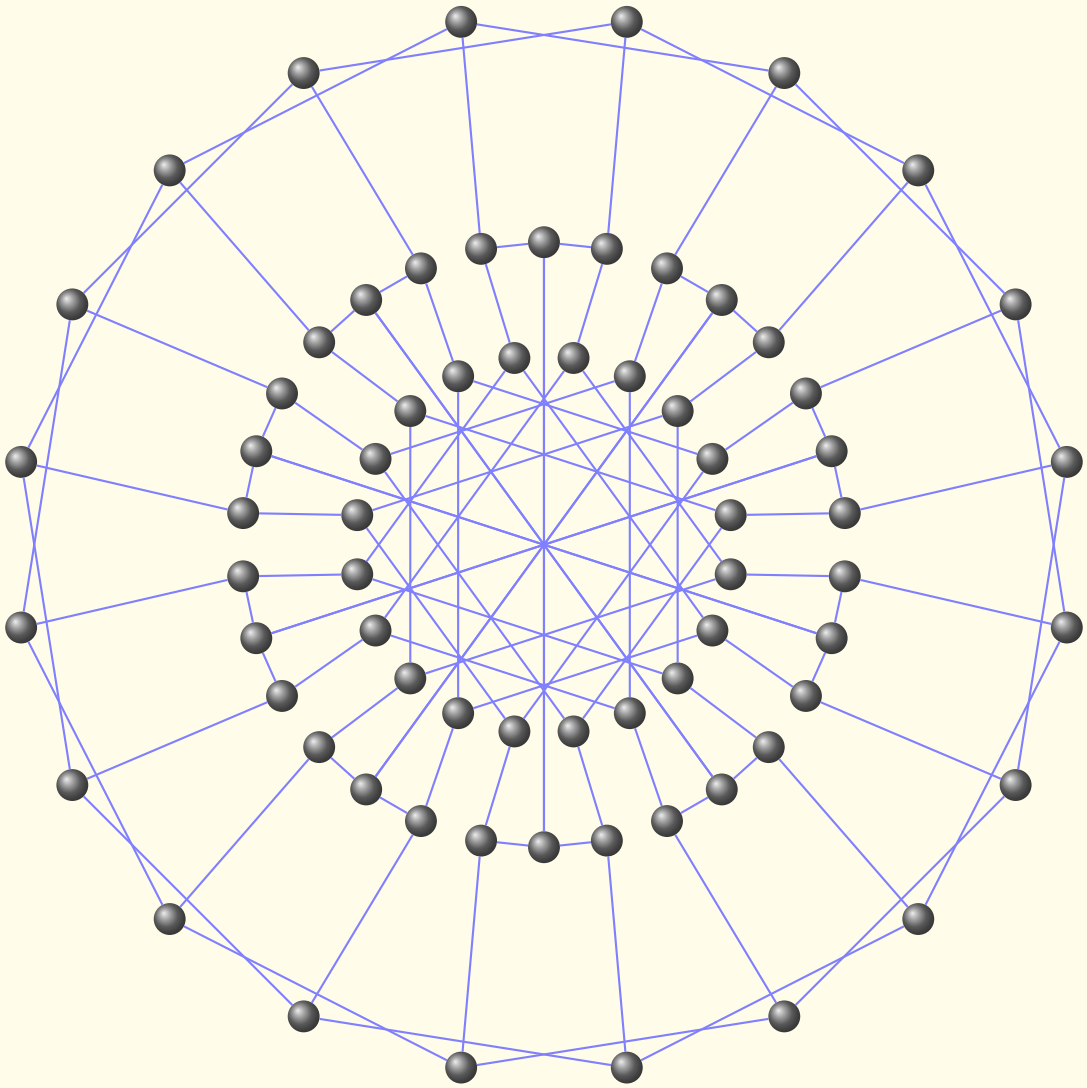
The Balaban 10-cage is one of the three(3,10)-cage graphs (Read 1998, p. 272). The Balaban (3,10)-cage was the first known example of a 10-cage (Balaban 1973; Pisanski 2001). Embeddings of all three possible (3,10)-cages (the others being the Harries graph and Harries-Wong graph) are given by Pisanski et al. (2001). Several embeddings are illustrated below, with the three rightmost being given by Pisanski and Randić (2000) It is a Hamiltonian graph and has Hamiltonian cycles. It has 1003 distinct LCF notations, with four of length two (illustrated above) and 999 of length 1. MathWorld by E.Weisstein

2.1 Balaban graph : first form



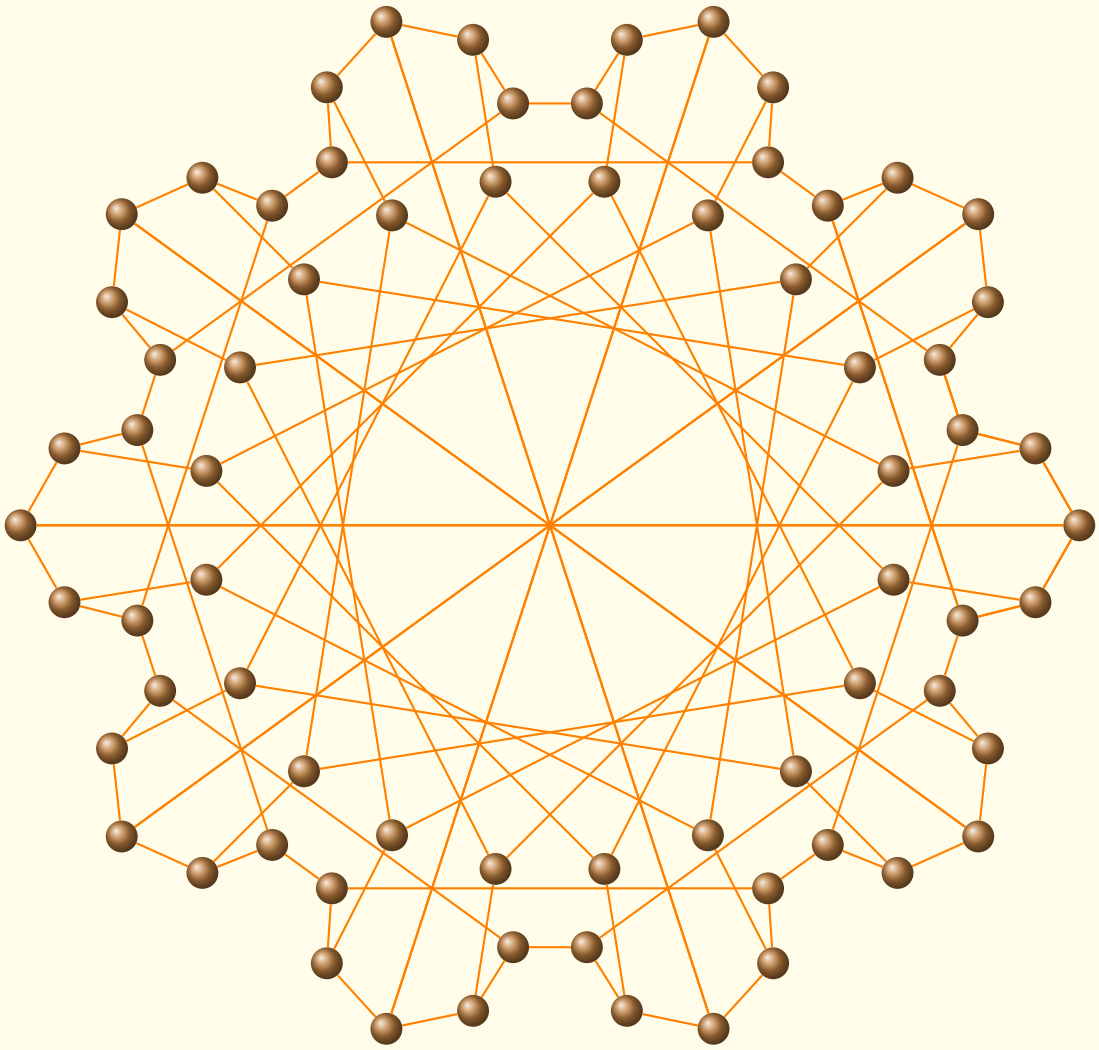
```
\begin{tikzpicture}[scale=.6]
 \GraphInit[vstyle=Art]
 \SetGraphArtColor{red}{olive}
\grBalaban[form=1,RA=7,RB=3,RC=3]
\end{tikzpicture}
```

2.2 Balaban graph : second form



```
\begin{tikzpicture}
 \GraphInit[vstyle=Art]
 \SetGraphArtColor{gray}{blue!50}
\grBalaban[form=2,RA=7,RB=7,RC=4,RD=2.5]
\end{tikzpicture}
```

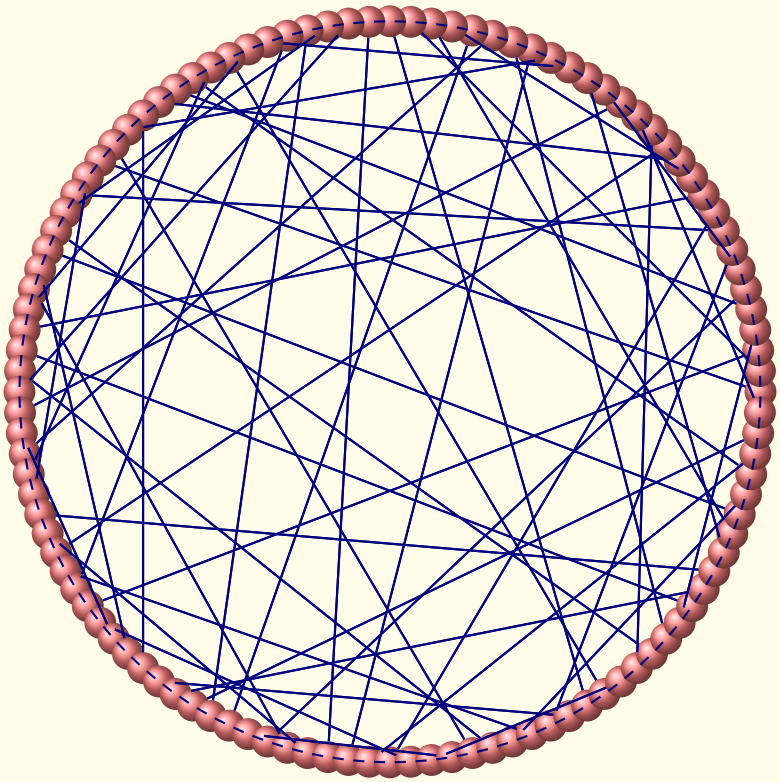
2.3 Balaban graph : third form



```
\begin{tikzpicture}
\GraphInit[vstyle=Art]
\SetGraphArtColor{brown}{orange}
\grBalaban[form=3,RA=7,RB=6.5,RC=5.6,RD=5.6,RE=4.6]
\end{tikzpicture}
```

2.4 Balaban graph : Balaban 11-Cage

The Balaban 11-cage is the unique 11-cage graph, discovered by Balaban (1973) and proven unique by McKay and Myrvold (2003). It has 112 vertices, 168 edges, girth 11 (by definition), diameter 8 and chromatic number 3.



```
\begin{tikzpicture}[scale=.7]
 \renewcommand*{\VertexInnerSep}{3pt}
 \renewcommand*{\VertexLineWidth}{0.4pt}
 \GraphInit[vstyle=Art]
 \SetGraphArtColor{red!50}{blue!50!black}
\grLCF[Math,RA=7]{%
44,26,-47,-15,35,-39,11,-27,38,-37,43,14,28,51,-29,-16,41,-11,%
-26,15,22,-51,-35,36,52,-14,-33,-26,-46,52,26,16,43,33,-15,%
 17,-53,23,-42,-35,-28,30,-22, 45,-44,16,-38,-16,50,-55,20,28,%
 -17,-43,47, 34,-26,-41,11,-36,-23,-16,41,17,-51,26,-33,47,17,%
 -11,-20 ,-30,21,29,36,-43,-52,10,39,-28,-17,-52,51,26,37,-17,%
 10,-10,-45,-34,17,-26,27,-21,46,53,-10,29,-50,35,15,-47,-29,-41,%
 26,33,55,-17,42,-26,-36,16}{1}
\end{tikzpicture}
```

SECTION 3

Complete BiPartite Graph

\grCompleteBipartite[⟨*options*⟩]{⟨*p*⟩}{⟨*q*⟩}

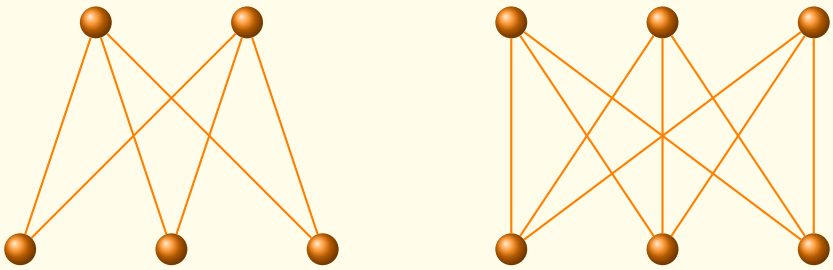
From MathWord : http://mathworld.wolfram.com/CompleteBipartiteGraph.html

A complete bipartite graph is a bipartite graph (i.e., a set of graph vertices decomposed into two disjoint sets such that no two graph vertices within the same set are adjacent) such that every pair of graph vertices in the two sets are adjacent. If there are p and q graph vertices in the two sets, the complete bipartite graph (sometimes also called a complete bigraph) is denoted $K_{p,q}$. The below figures show $K_{3,2}$ and $K_{3,3}$. $K_{3,3}$ is also known as the utility graph (and the circulant graph $Ci_{1,3}(6)$), and is the unique 4-cage graph. MathWorld by E.Weisstein

From Wikipedia : http://en.wikipedia.org/wiki/Complete_bipartite_graph

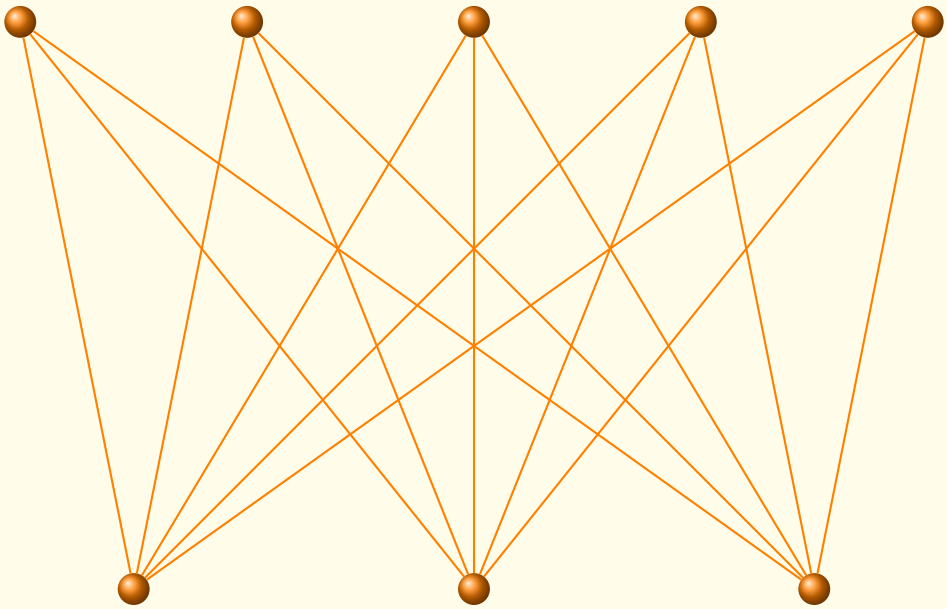
In the mathematical field of graph theory, a complete bipartite graph or biclique is a special kind of bipartite graph where every vertex of the first set is connected to every vertex of the second set. the graph $K_{1,3}$ is also called a claw.

3.1 Complete bipartite graphs $K_{3,2}$ and $K_{3,3}$



```
\begin{tikzpicture}
  \GraphInit[vstyle=Art]
  \grCompleteBipartite[RA=2,RB=2,RS=3]{3}{2}
 \end{tikzpicture}\hspace*{2cm}
 \begin{tikzpicture}
  \GraphInit[vstyle=Art]
  \grCompleteBipartite[RA=2,RB=2,RS=3]{3}{3}
\end{tikzpicture}
```

3.2 Complete bipartite graphs $K_{3,5}$

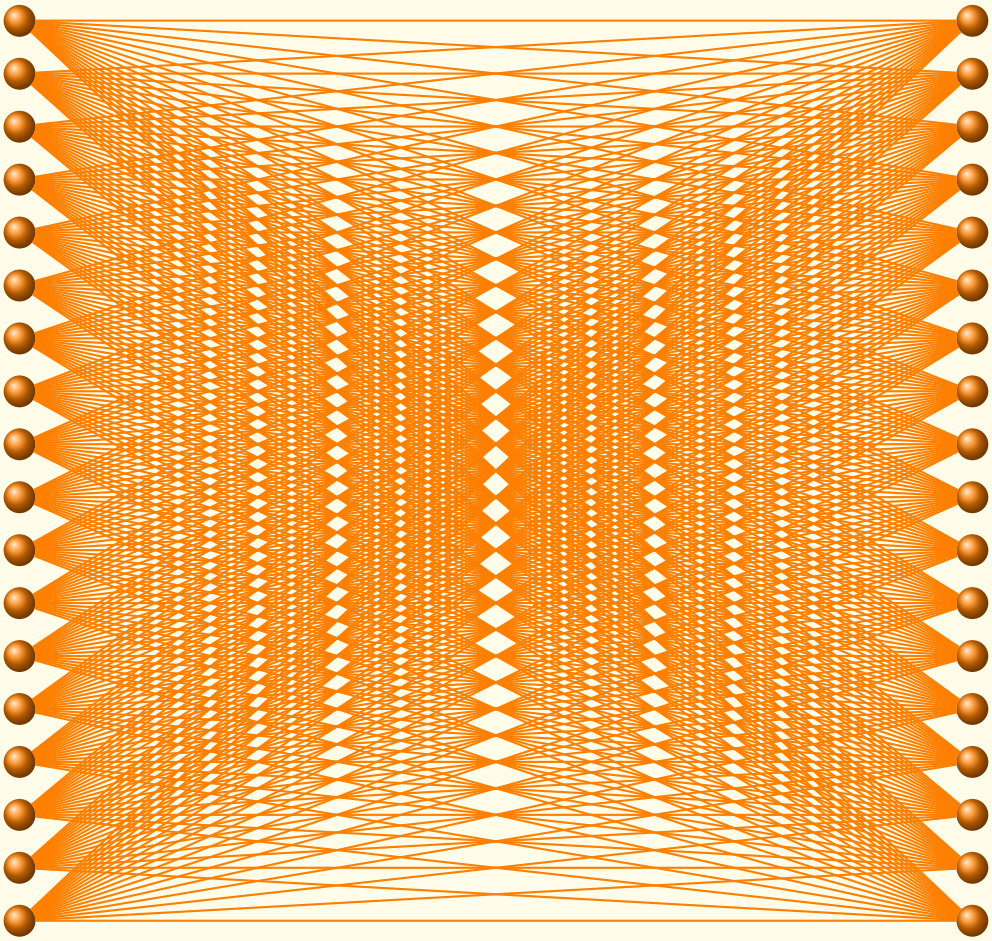


```
\begin{tikzpicture}[scale=1.5]
  \GraphInit[vstyle=Art]
 \grCompleteBipartite[RA=3,RB=2,RS=5]{3}{5}
\end{tikzpicture}
```

3.3 Complete bipartite graph : $K_{18,18}$

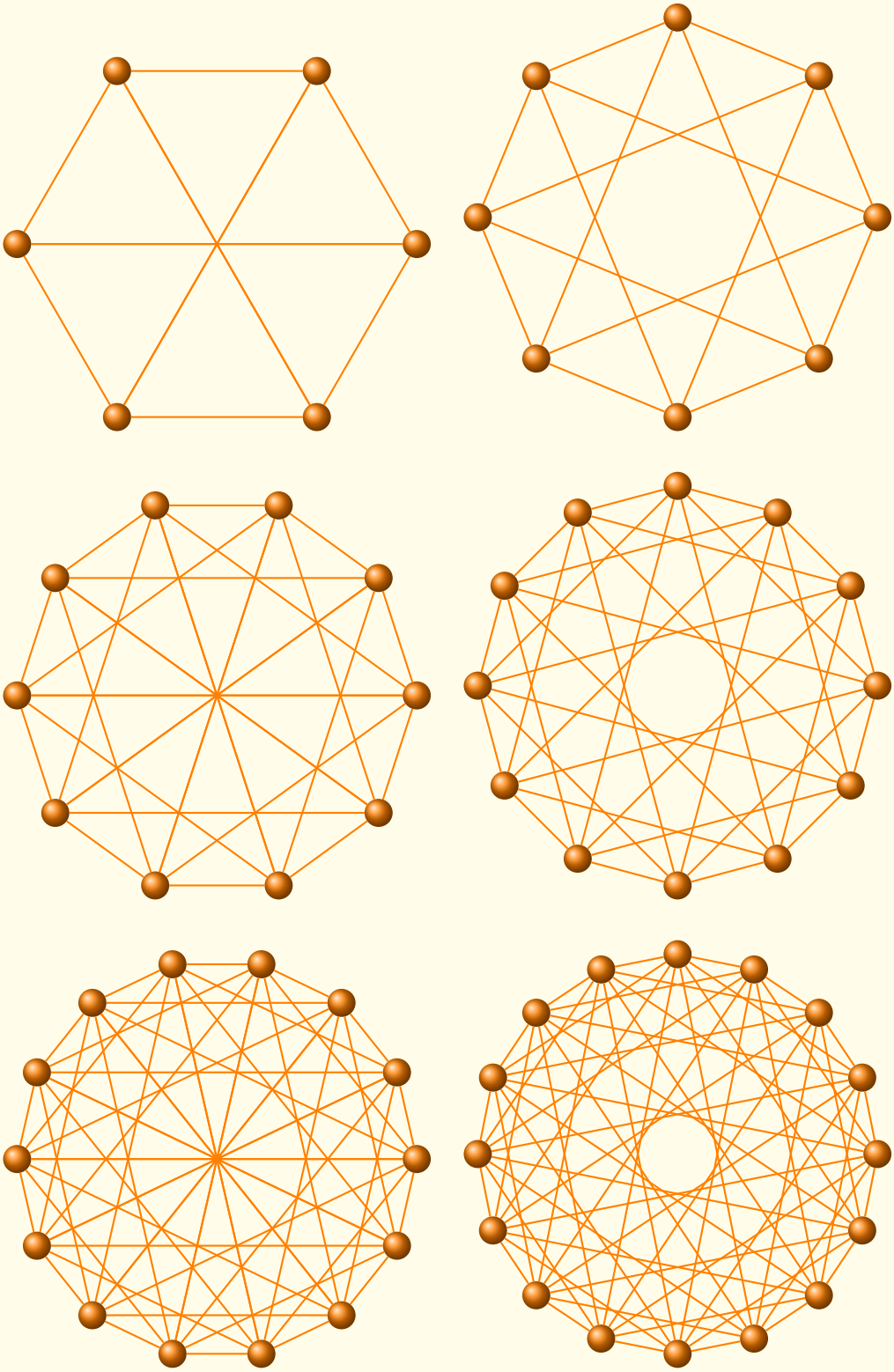
The complete bipartite graph illustrated below plays an important role in the novel Foucault's Pendulum by Umberto Eco.

MathWorld by E.Weisstein



```
\begin{tikzpicture}[rotate=90,scale=1.4]
  \GraphInit[vstyle=Art]
  \grCompleteBipartite[RA=0.5,RB=0.5,RS=9]{18}{18}
 \end{tikzpicture}
```

A complete bipartite graph $K_{n,n}$ is a circulant graph (if the order is equal to $2n$ then $L = 1,3,\ldots,n$). The code is on the next page

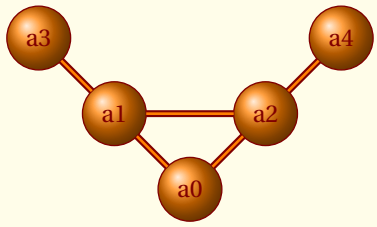


```
\begin{tikzpicture}
  \GraphInit[vstyle=Art]
  \grCirculant[RA=3]{6}{1,3}
\end{tikzpicture}\hspace*{12pt}
\begin{tikzpicture}
  \GraphInit[vstyle=Art]
  \grCirculant[RA=3]{8}{1,3}
\end{tikzpicture}

\vspace*{12pt}
\begin{tikzpicture}
  \GraphInit[vstyle=Art]
  \grCirculant[RA=3]{10}{1,3,5}
\end{tikzpicture}\hspace*{12pt}
\begin{tikzpicture}
  \GraphInit[vstyle=Art]
  \grCirculant[RA=3]{12}{1,3,5}
\end{tikzpicture}

\vspace*{12pt}
\begin{tikzpicture}
       \GraphInit[vstyle=Art]
\grCirculant[RA=3]{14}{1,3,5,7}
\end{tikzpicture}\hspace*{12pt}
\begin{tikzpicture}
       \GraphInit[vstyle=Art]
\grCirculant[RA=3]{16}{1,3,5,7}
\end{tikzpicture}
```

SECTION 4

Bull

The bull graph, 5 vertices, 5 edges, resembles to the head of a bull if drawn properly. The bull graph is a simple graph on 5 nodes and 5 edges whose name derives from its resemblance to a schematic illustration of a bull




```
\begin{tikzpicture}[node distance=4cm]
  \GraphInit[vstyle=Shade]
  \Vertex{a0}
  \NOEA(a0){a2}
  \NOEA(a2){a4}
  \NOWE(a0){a1}
  \NOWE(a1){a3}
  \Edges(a0,a1,a3)
  \Edges(a0,a2,a4)
  \Edge(a1)(a2)
\end{tikzpicture}
```

SECTION 5

Cage

\Cage Graphs

From Wikipedia http://en.wikipedia.org/wiki/Cage_(graph_theory)
In the mathematical area of graph theory, a cage is a regular graph that has as few vertices as possible for its girth.
Formally, an (r,g)-graph is defined to be a graph in which each vertex has exactly r neighbors, and in which the shortest cycle has length exactly g. It is known that an (r,g)-graph exists for any combination of $r \geq 2$ and $g \geq 3$. An (r,g)-cage is an (r,g)-graph with the fewest possible number of vertices, among all (r,g)-graphs.

From MathWorld http://mathworld.wolfram.com/CageGraph.html
A (r,g)-cage graph is a v-regular graph of girth g having the minimum possible number of nodes. When v is not explicitly stated, the term "g-cage" generally refers to a $(3,g)$-cage. MathWorld by E.Weisstein

Examples :

(r,g)	Names
$(3,3)$	complete graph K_4
$(3,4)$	complete bipartite graph $K_{3,3}$ Utility Graph3
$(3,5)$	Petersen graph 25
$(3,6)$	Heawood graph 18
$(3,7)$	McGee graph 22
$(3,8)$	Levi graph 21
$(3,10)$	Balaban 10-cage 2
$(3,11)$	Balaban 11-cage 2
$(3,12)$	Tutte 12-cage
$(4,3)$	complete graph K_5
$(4,4)$	complete bipartite graph $K_{4,4}$ 3
$(4,5)$	Robertson graph27
$(4,6)$	Wong (1982)29

SECTION 6

Cocktail Party graph

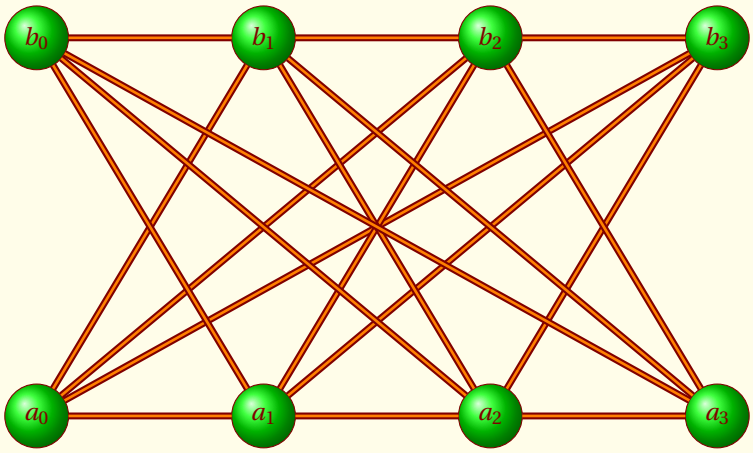
\grCocktailParty[⟨*options*⟩]{⟨*integer*⟩}

From MathWord : http://mathworld.wolfram.com/CocktailPartyGraph.html

The cocktail party graph of order , also called the hyperoctahedral graph (Biggs 1993, p. 17) is the graph consisting of two rows of paired nodes in which all nodes but the paired ones are connected with a graph edge. It is the graph complement of the ladder graph , and the dual graph of the hypercube graph.
This graph arises in the handshake problem. It is a complete n-partite graph that is denoted by Brouwer et al. (1989, pp. 222-223), and is distance-transitive, and hence also distance-regular.
The cocktail party graph of order is isomorphic to the circulant graph. MathWorld by E.Weisstein

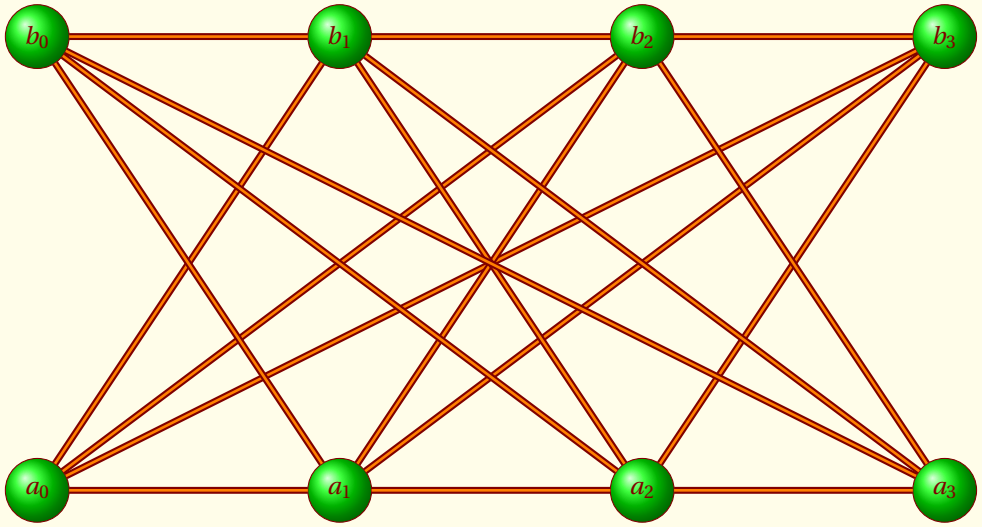
The Chvátal graph is implemented in **tkz-berge** as **\grCocktailParty** with two forms.

6.1 Cocktail Party graph form 1




```
    \begin{tikzpicture}
      \grCocktailParty[RA=3,RS=5]{4}
\end{tikzpicture}
```

6.2 Cocktail Party graph form 2




```
\begin{tikzpicture}
   \grCocktailParty[form=2,RA=4,RS=6]{4}
 \end{tikzpicture}
```

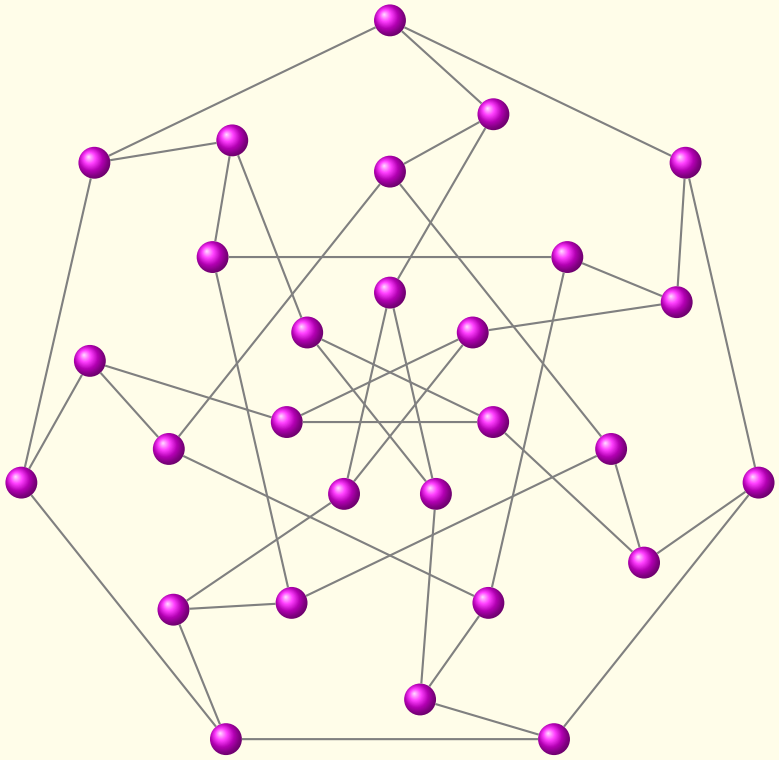
SECTION 7

Coxeter

From MathWorld : http://mathworld.wolfram.com/CoxeterGraph.html

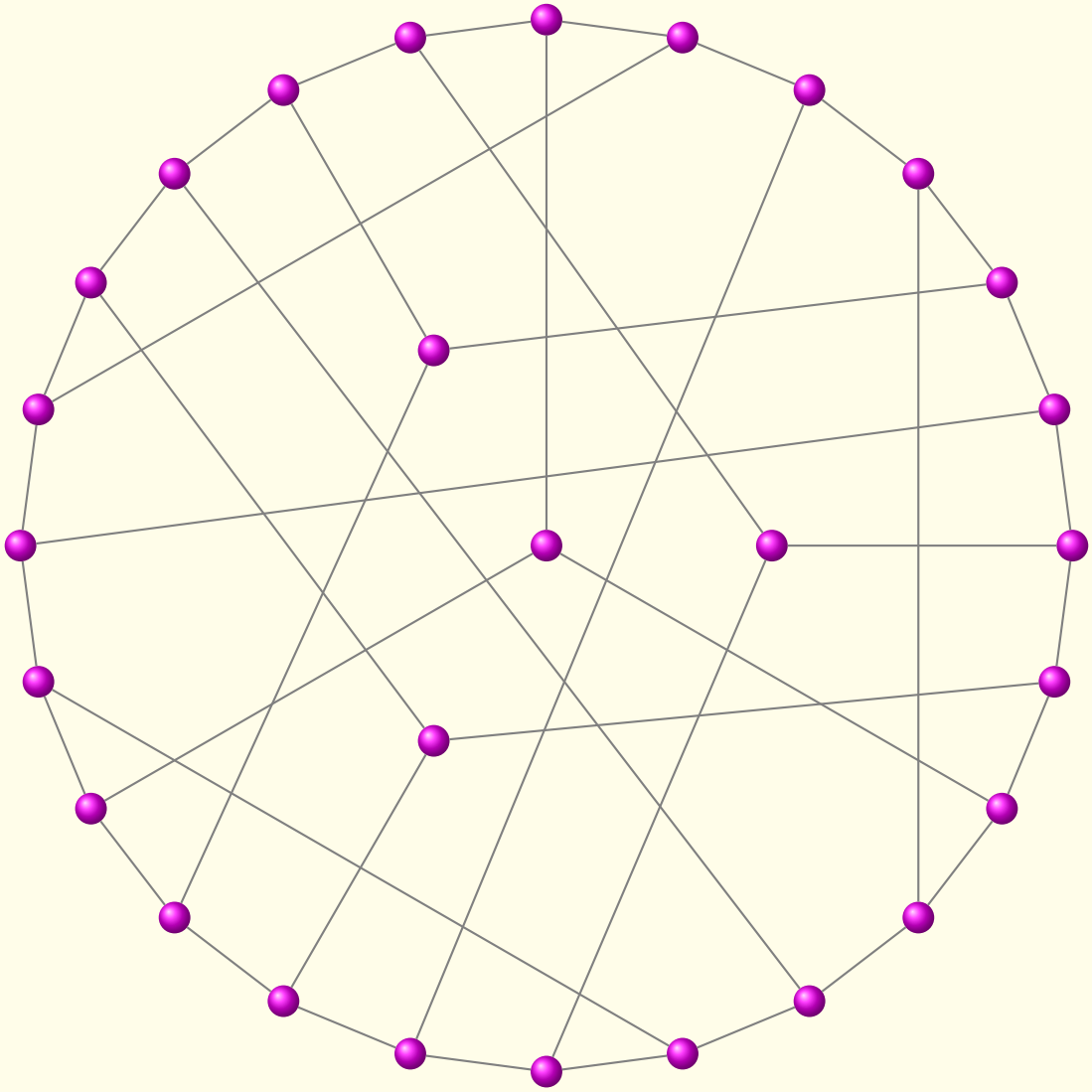
The Coxeter graph is a nonhamiltonian cubic symmetric graph on 28 vertices and 42 edges.

7.1 Coxeter graph I



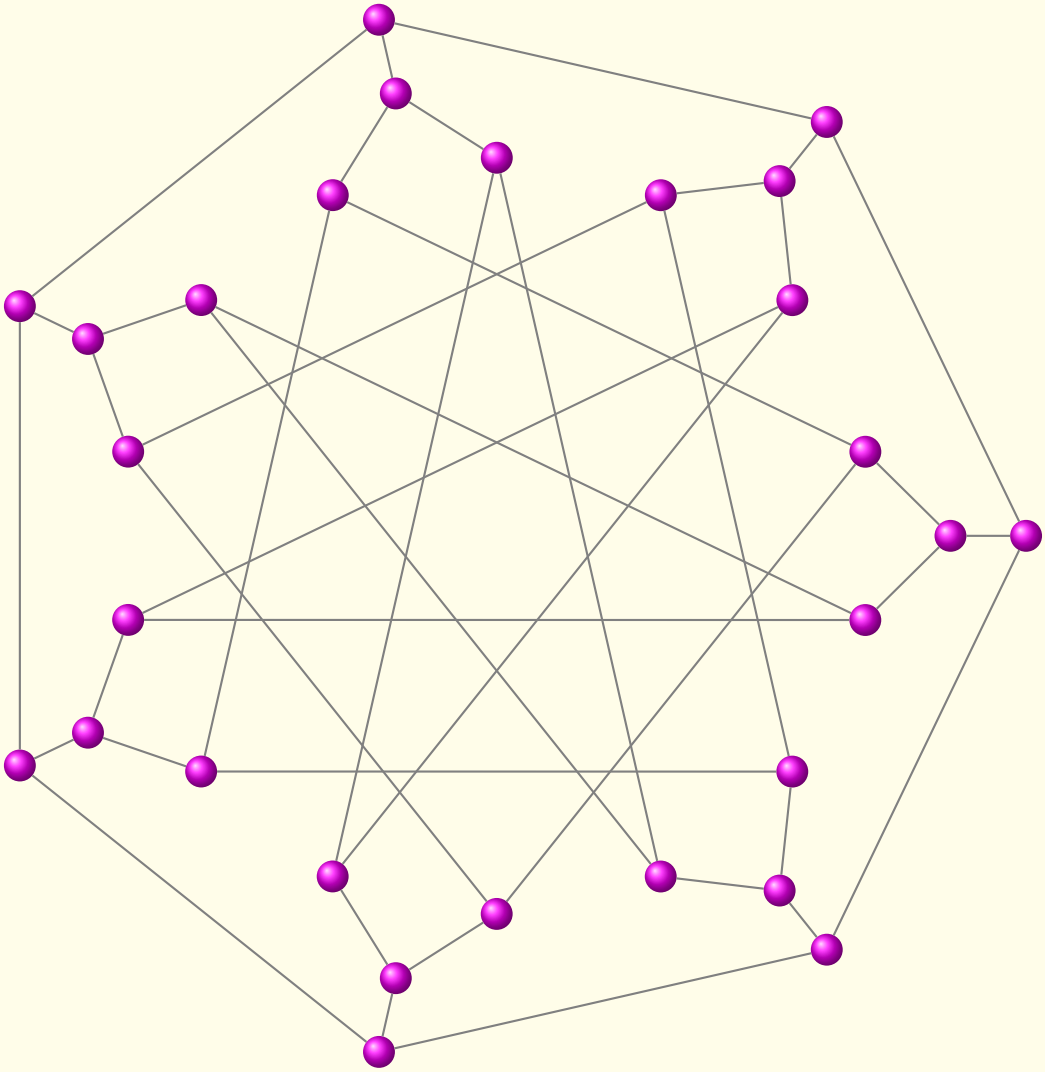
```
\begin{tikzpicture}[rotate=90,scale=1]
  \GraphInit[vstyle=Art]
  \SetGraphArtColor{magenta}{gray}
  \grCycle[RA=5,prefix=a]{7}
  \begin{scope}[rotate=-20]\grEmptyCycle[RA=4,prefix=b]{7}\end{scope}
  \grCirculant[RA=3,prefix=c]{7}{2}
  \grCirculant[RA=1.4,prefix=d]{7}{3}
  \EdgeIdentity{a}{b}{7}
  \EdgeIdentity{b}{c}{7}
  \EdgeIdentity{b}{d}{7}
\end{tikzpicture}
```

7.2 Coxeter graph II



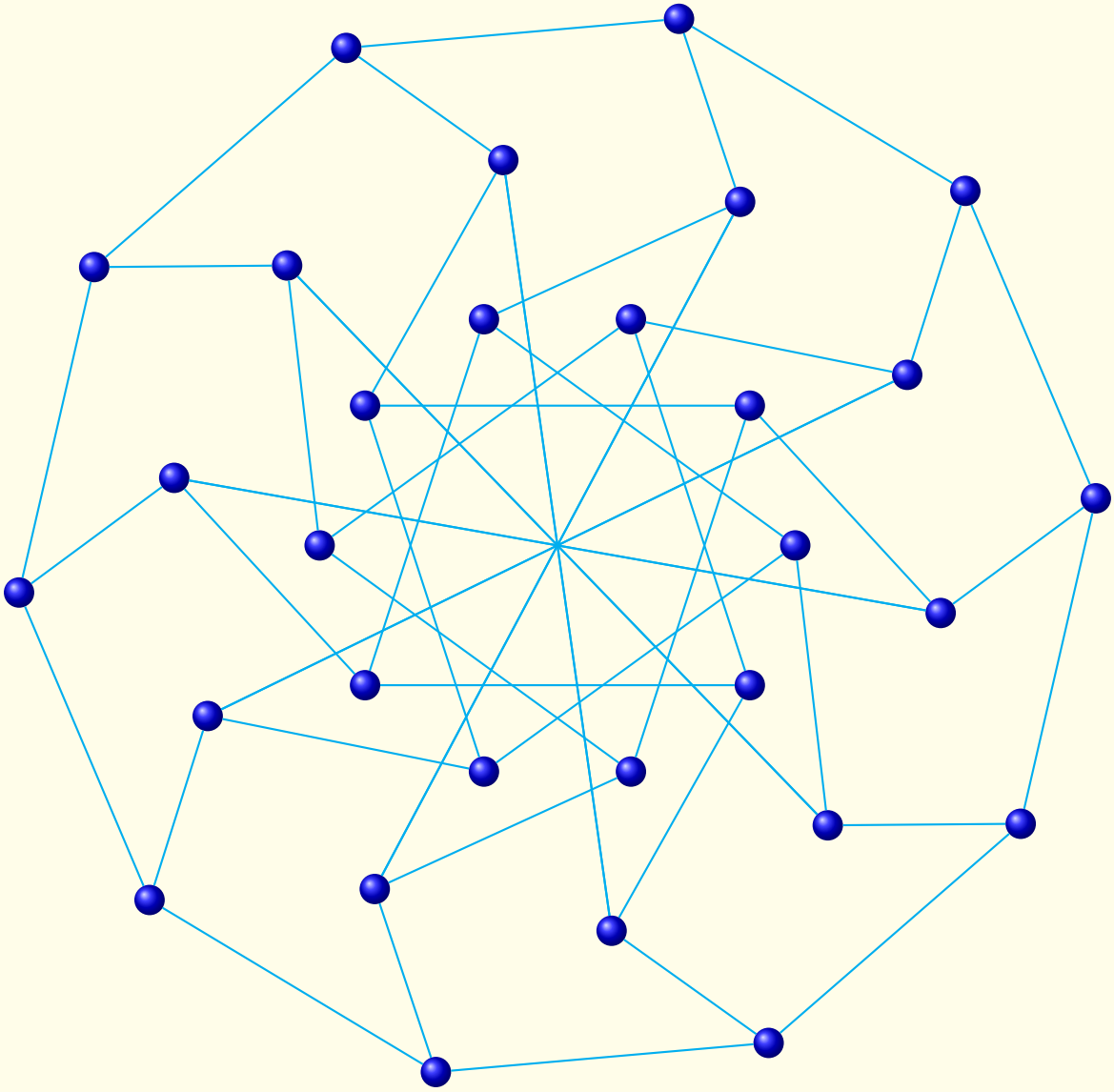
```
\begin{tikzpicture}
  \GraphInit[vstyle=Art]
  \SetGraphArtColor{magenta}{gray}
  \grCycle[RA=7,prefix=b]{24}
  \grEmptyStar[RA=3,prefix=a]{4}
  \EdgeDoubleMod{a}{3}{0}{1}{b}{24}{0}{8}{2}
  \EdgeDoubleMod{a}{3}{0}{1}{b}{24}{7}{8}{2}
  \EdgeDoubleMod{a}{3}{0}{1}{b}{24}{18}{8}{2}
  \EdgeDoubleMod{a}{4}{3}{0}{b}{24}{22}{8}{2}
  \EdgeInGraphMod*{b}{24}{6}{5}{8}
  \EdgeInGraphMod*{b}{24}{11}{1}{8}
\end{tikzpicture}
```

7.3 Coxeter graph III



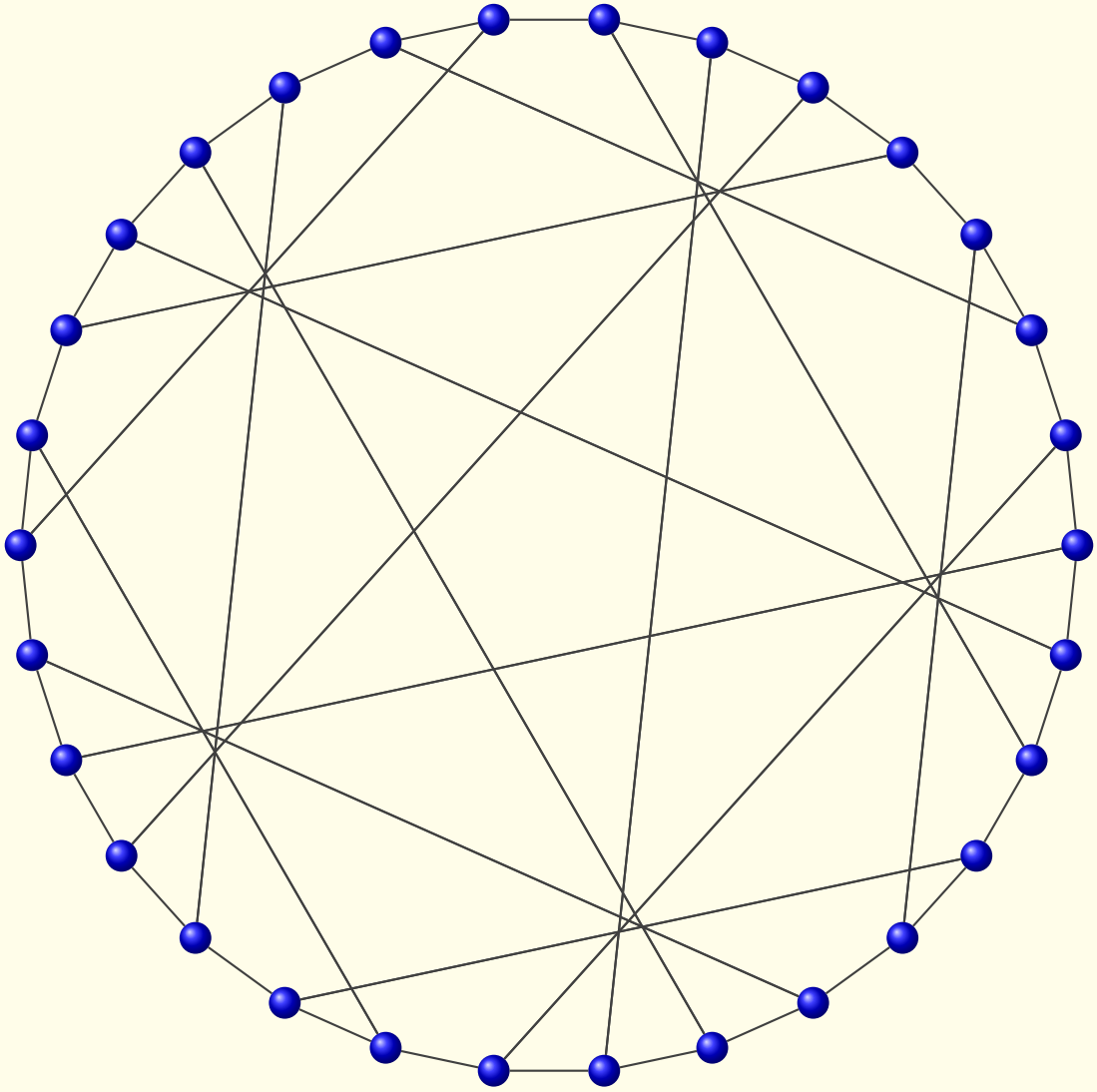
```
\begin{tikzpicture}
  \GraphInit[vstyle=Art]
  \SetGraphArtColor{magenta}{gray}
  \grCycle[RA=7,prefix=c]{7}
  \grEmptyCycle[RA=6,prefix=b]{7}
  \begin{scope}[rotate=12.85]\grEmptyCycle[RA=5,prefix=a]{14}\end{scope}
  \EdgeIdentity{b}{c}{7}
  \EdgeDoubleMod{b}{7}{0}{1}{a}{14}{0}{2}{6}
  \EdgeDoubleMod{b}{7}{0}{1}{a}{14}{13}{2}{6}
  \EdgeInGraphModLoop{a}{14}{4}{0}{0}
  \EdgeInGraphModLoop{a}{14}{6}{1}{1}
\end{tikzpicture}
```

7.4 Tutte-Coxeter graph I



```
\begin{tikzpicture}[scale=3]
  \GraphInit[vstyle=Art]
  \SetGraphArtColor{blue}{cyan}
  \begin{scope}[rotate=5]\grCycle[RA=2.5,prefix=a]{10}\end{scope}
  \begin{scope}[rotate=-10]\grCirculant[RA=1.8,prefix=b]{10}{5}\end{scope}
  \begin{scope}[rotate=36]\grCirculant[RA=1.1,prefix=c]{10}{3}\end{scope}
  \EdgeIdentity{a}{b}{10}
  \EdgeIdentity{b}{c}{10}
\end{tikzpicture}
```

7.5 Tutte-Coxeter graph II



```
\begin{tikzpicture}
     \GraphInit[vstyle=Art]
     \SetGraphArtColor{blue}{darkgray}
     \grLCF[RA=7]{-13,-9,7,-7,9,13}{5}
\end{tikzpicture}
```

SECTION 8

Chvatal

\grChvatal[⟨*options*⟩]

From Wikipedia : http://en.wikipedia.org/wiki/Václav_Chvátal

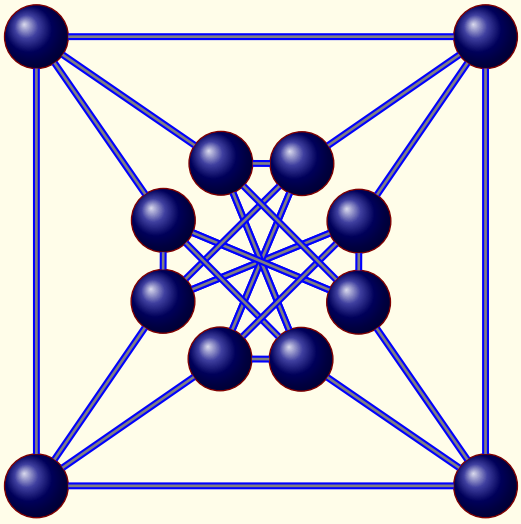
Chvátal first learned of graph theory in 1964, on finding a book by Claude Berge in a Pilsen bookstore, and his first mathematical publication, at the age of 19, concerned directed graphs that cannot be mapped to themselves by any nontrivial graph homomorphism.
Gallery Theorem—which determines the number of guards required to survey the walls of a polygonal art gallery (and has prompted much research), and constructed the smallest triangle-free 4-chromatic 4-regular graph, a beautiful graph now known as the Chvatal graph.

From MathWord : http://mathworld.wolfram.com/ChvatalGraph.html

The Chvátal graph is a quartic graph on 12 nodes and 24 edges. It has chromatic number 4, and girth 4.
MathWorld by E.Weisstein

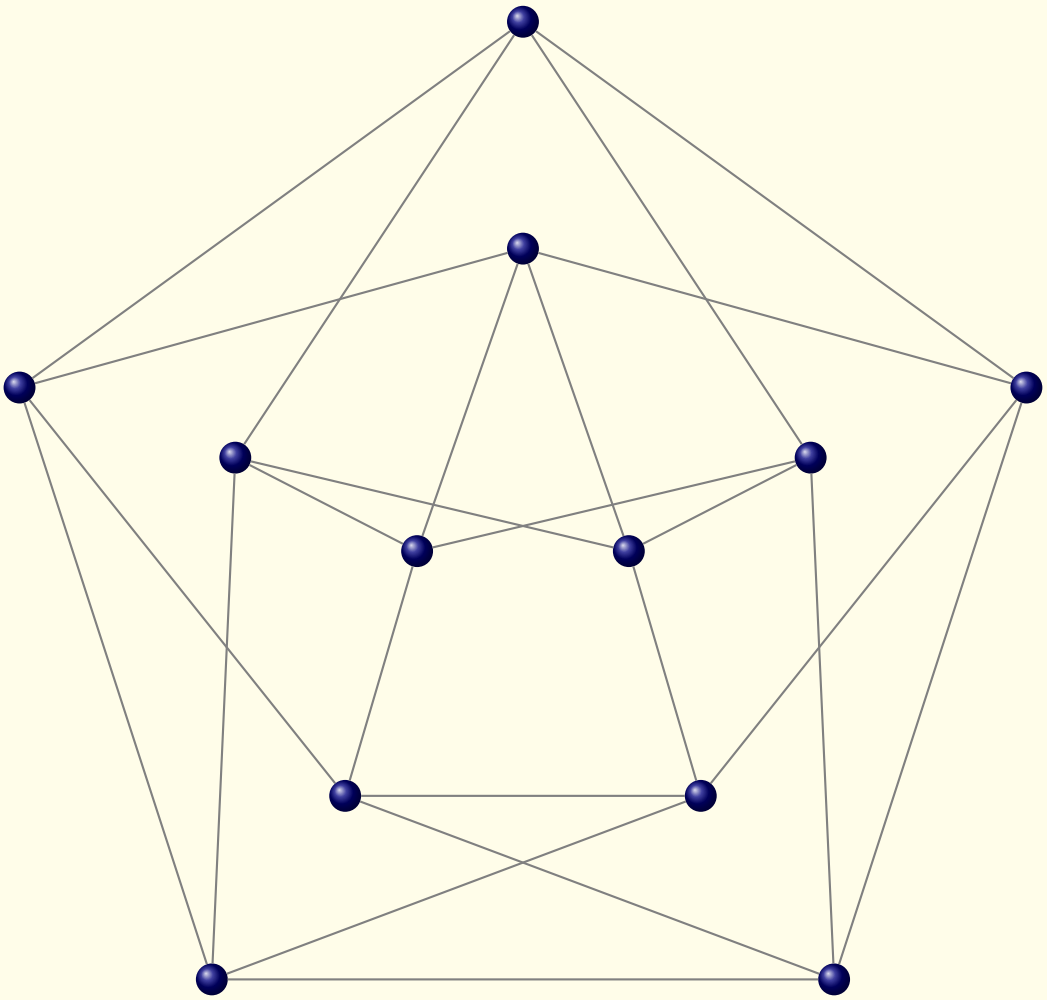
The Chvátal graph is implemented in **tkz-berge** as **\grChvatal** with three forms.

8.1 Chvatal graph I



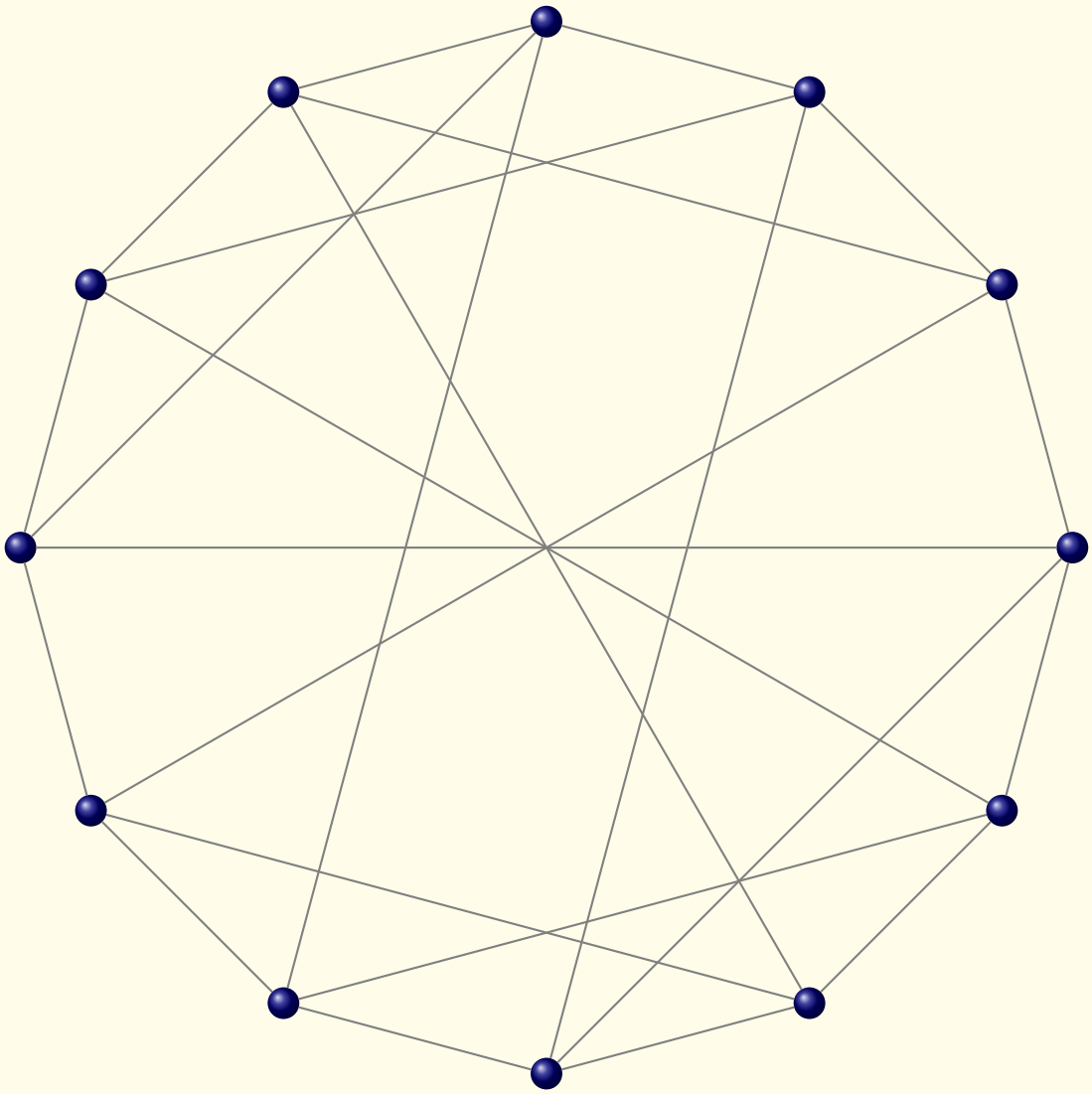
```
\begin{tikzpicture}[scale=.7]
   \GraphInit[vstyle=Shade]
   \SetVertexNoLabel
   \SetGraphShadeColor{blue!50!black}{blue}{gray}
   \grChvatal[RA=6,RB=2]
\end{tikzpicture}
```

8.2 Chvatal graph II



```
\begin{tikzpicture}
  \GraphInit[vstyle=Art]
  \SetGraphArtColor{blue!50!black}{gray}
  \grChvatal[form=2,RA=7,RB=4,RC=1.4]
\end{tikzpicture}
```

8.3 Chvatal graph III



```
\begin{tikzpicture}
 \GraphInit[vstyle=Art]
 \SetGraphArtColor{blue!50!black}{gray}
 \grChvatal[form=3,RA=7]
\end{tikzpicture}
```

SECTION 9

Crown

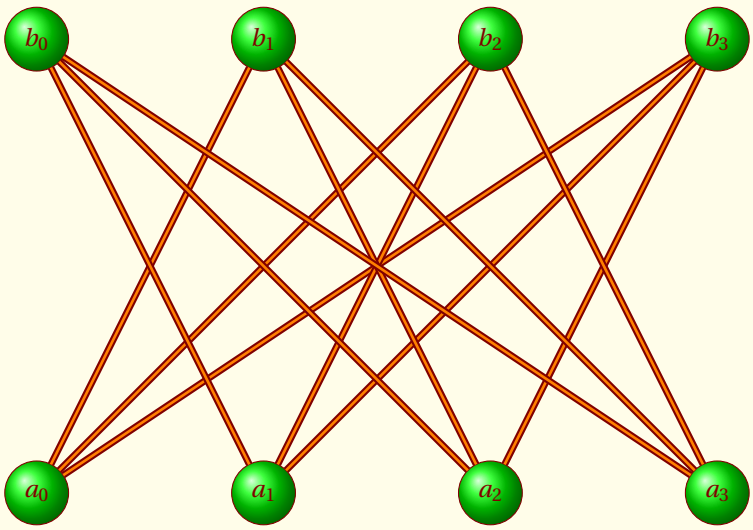
\grCrown[⟨*options*⟩]{⟨*integer*⟩}

From MathWord : http://mathworld.wolfram.com/CrownGraph.html

The Crown graph for an integer is the graph with vertex set $\{x_0, x_1, \ldots, x_{n-1}, y_0, y_1, \ldots, y_{n-1}\}$ *and edge set*
$\{(x_i, x_j) : 0 \leq i, j \leq n-1, i \neq j\}$. MathWorld by E.Weisstein

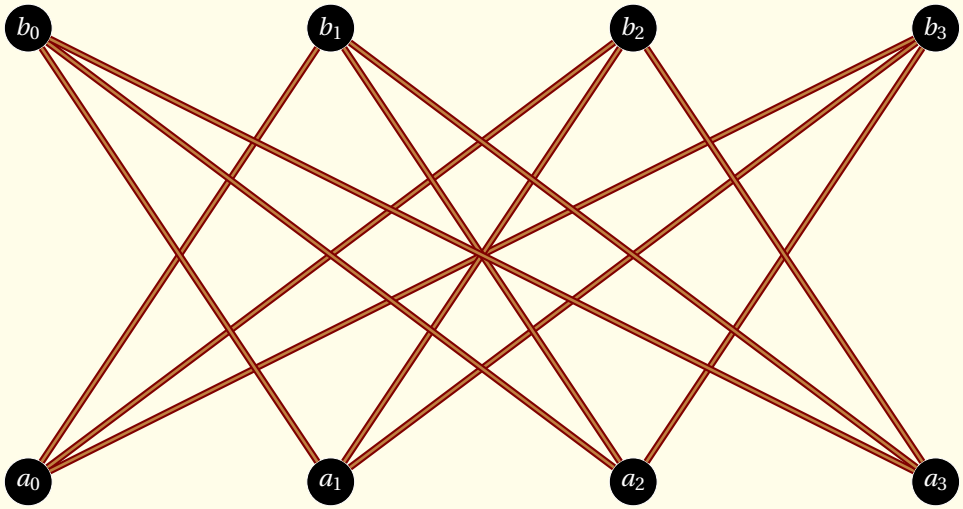
The Crown graph is implemented in **tkz-berge** as **\grCrown** with two forms.

9.1 Crown graph form 1




```
\begin{tikzpicture}
\tikzstyle{VertexStyle}   = [shape           = circle,
                             shading         = ball,
                             ball color      = green,
                             minimum size    = 24pt,
                             draw]
\tikzstyle{EdgeStyle}     = [thick,
                             double          = orange,
                             double distance = 1pt]
\SetVertexLabel\SetVertexMath
\grCrown[RA=3,RS=6]{4}
\end{tikzpicture}
```

9.2 Crown graph form 2




```
\begin{tikzpicture}
  \grCrown[form=2,RA=4,RS=6]{4}
\end{tikzpicture}
```

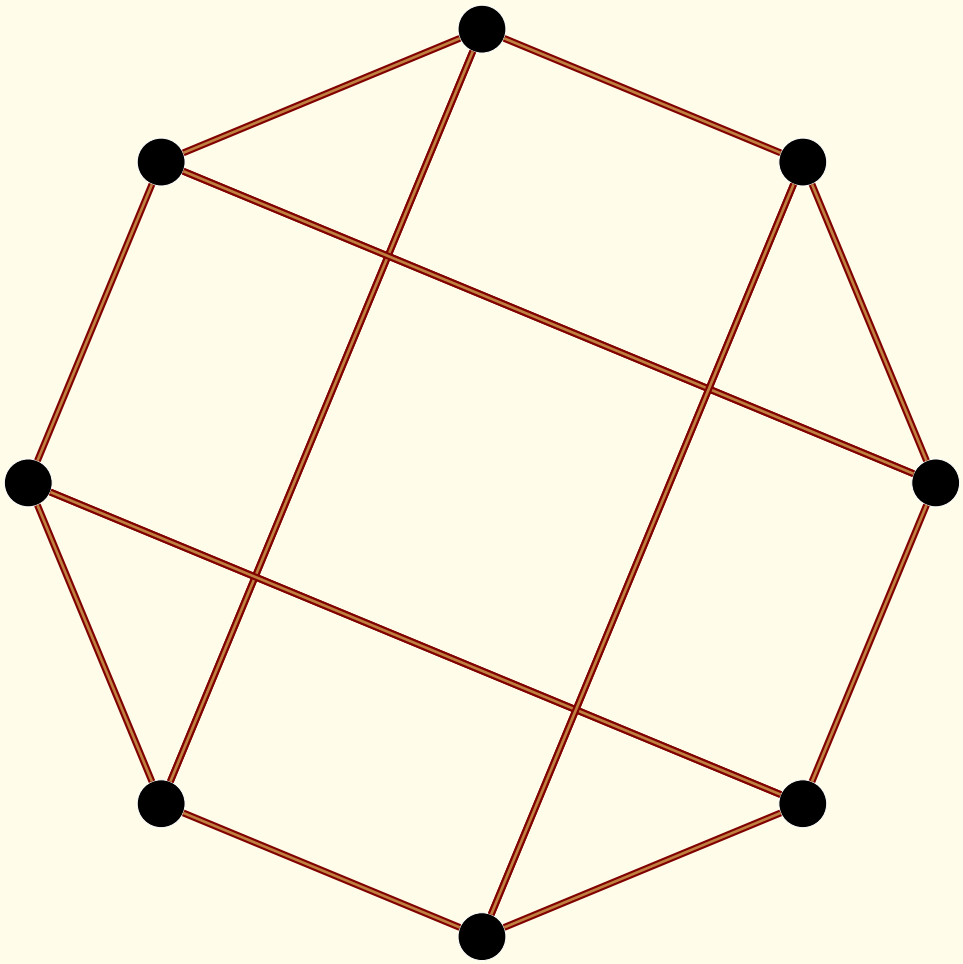
SECTION 10

Cubic Symmetric Graphs

A cubic symmetric graph is a symmetric cubic (i.e., regular of order 3). Such graphs were first studied by Foster (1932). They have since been the subject of much interest and study. Since cubic graphs must have an even number of vertices, so must cubic symmetric graphs.

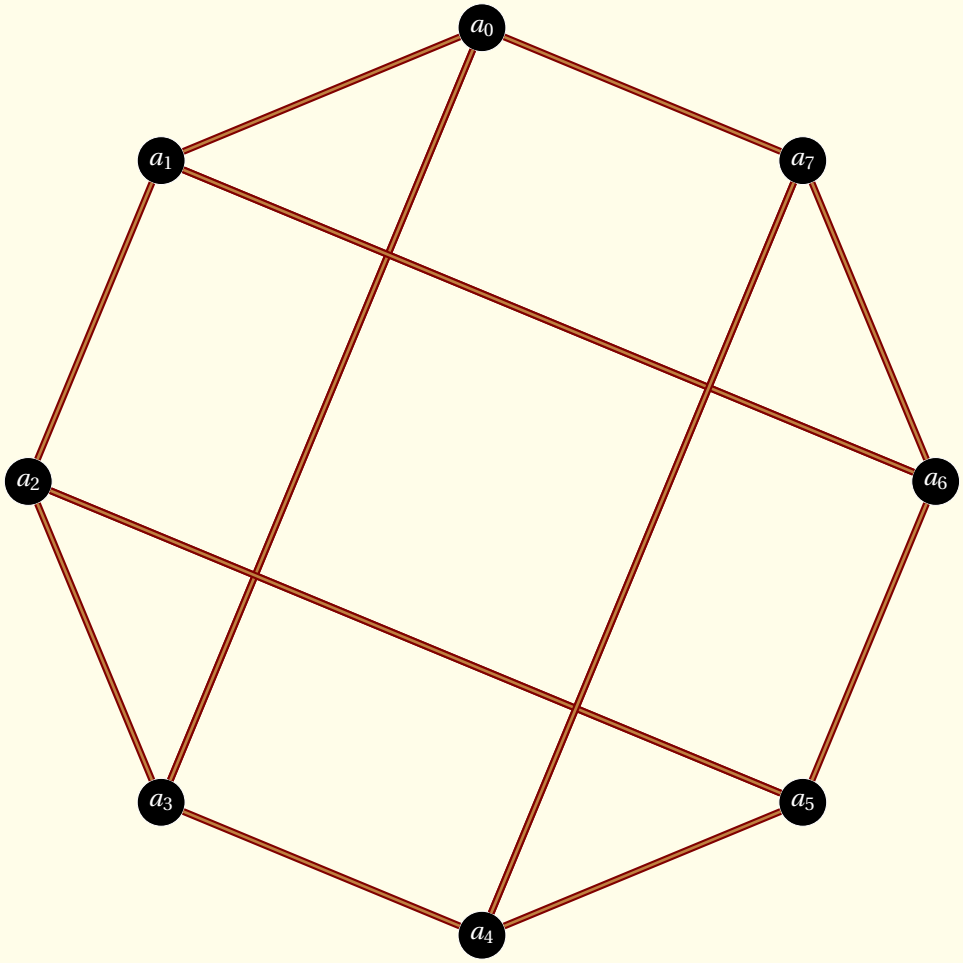
The circulant graph , is illustrated below.

10.1 Cubic Symmetric Graph form 1



```
\begin{tikzpicture}[rotate=90]
     \SetVertexNoLabel
     \grLCF[RA=6]{3,-3}{4}
\end{tikzpicture}
```

10.2 Cubic Symmetric Graph form 2




```
\begin{tikzpicture}[rotate=90]
  \tikzstyle{VertexStyle} = [shape               =  circle,%
                             color               =  white,
                             fill                =  black,
                             very thin,
                             inner sep           =  0pt,%
                             minimum size        =  18pt,
                             draw]
  \tikzstyle{EdgeStyle}   = [thick,%
                             double              = brown,%
                             double distance     = 1pt]
  \grLCF[Math,RA=6]{3,-3}{4}
\end{tikzpicture}
```

SECTION 11

Desargues

`\grDesargues[⟨options⟩]`

From Wikipedia : http://en.wikipedia.org/wiki/Desargues_graph

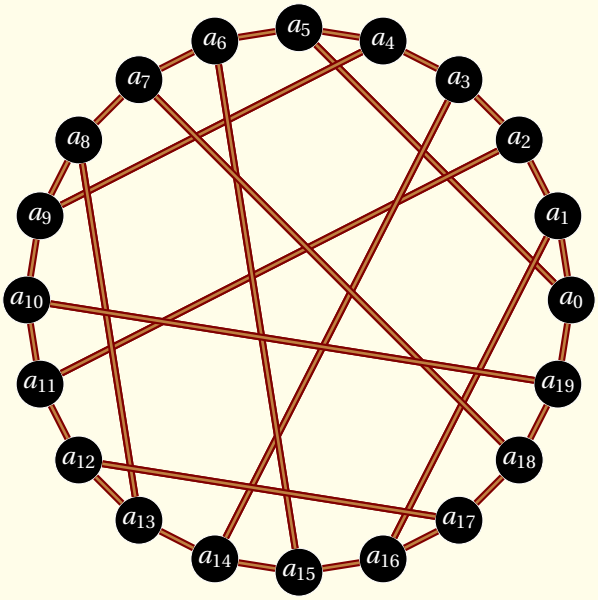
In the mathematical field of graph theory, the Desargues graph is a 3-regular graph with 20 vertices and 30 edges, formed as the Levi graph of the Desargues configuration.The Desargues graph can also be formed as a double cover of the Petersen graph, as the generalized Petersen graph G(10,3), or as the bipartite Kneser graph $H_{5,2}$.

From MathWord : http://mathworld.wolfram.com/DesarguesGraph.html

The Desargues graph is a cubic symmetric graph distance-regular graph on 20 vertices and 30 edges, illustrated above in several embeddings. It can be represented in LCF notation as (Frucht 1976) and is isomorphic to the bipartite Kneser graph . It is the incidence graph of the Desargues configuration. MathWorld by E.Weisstein

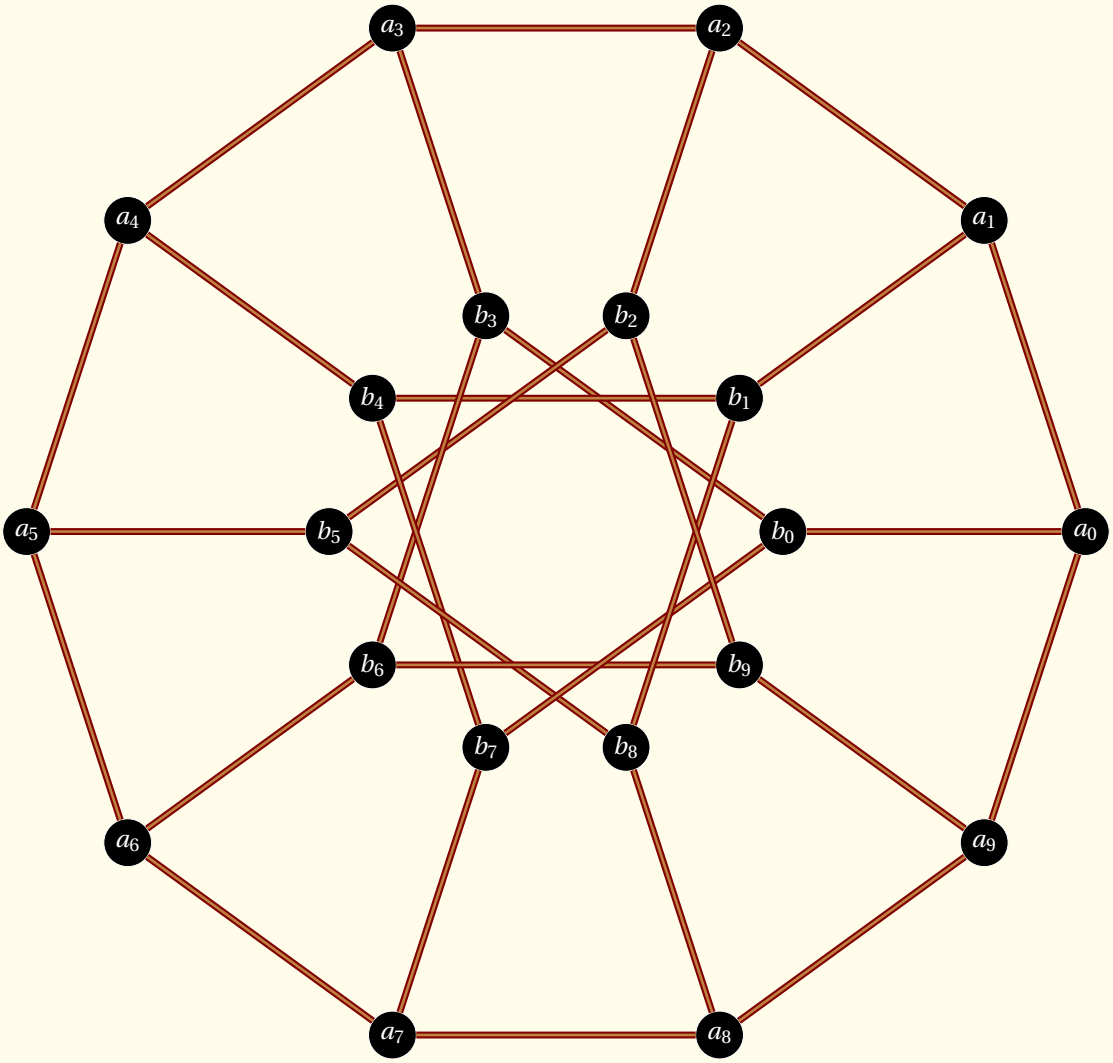
The Desargues graph is implemented in **tkz-berge** as **\grDesargues** with two forms.

11.1 The Desargues graph : form 1



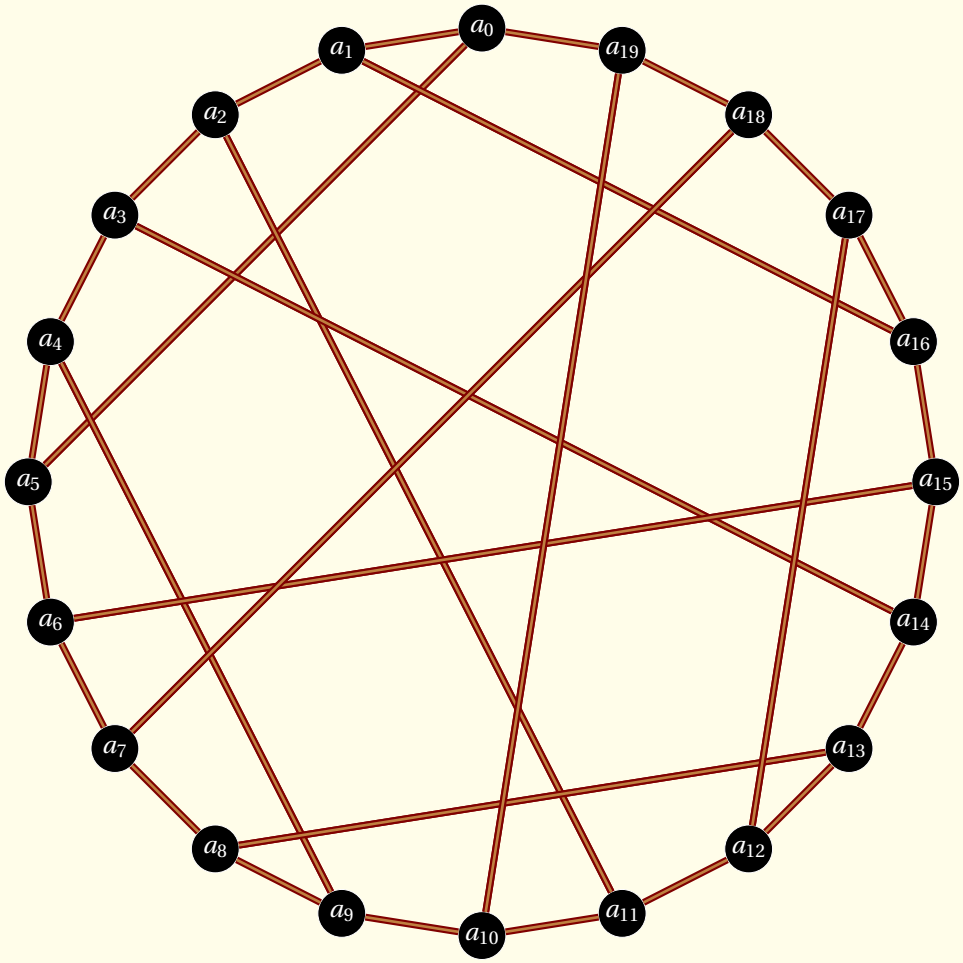

```
\begin{tikzpicture}[scale=.6]
    \grDesargues[Math,RA=6]
\end{tikzpicture}
```

11.2 The Desargues graph : form 2



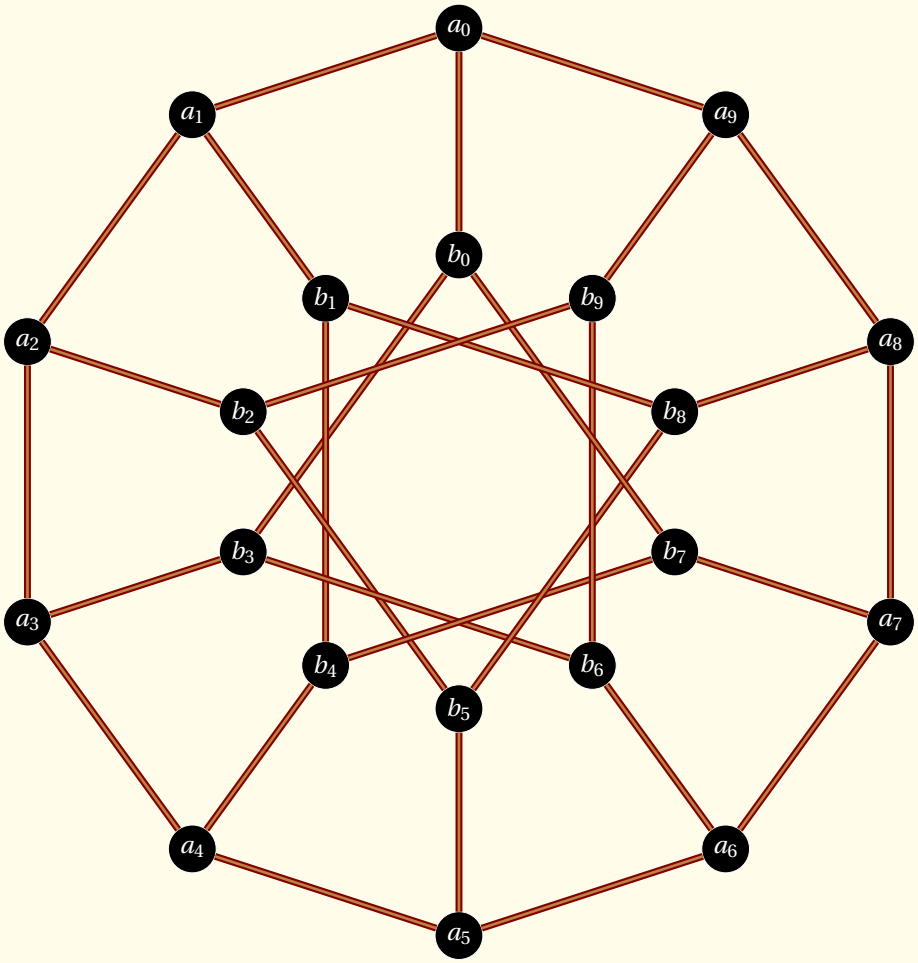

```
\begin{tikzpicture}
   \grDesargues[form=2,Math,RA=7]
\end{tikzpicture}
```

11.3 The Desargues graph wth LCF notation




```
\begin{tikzpicture}[rotate=90]
   \grLCF[Math,RA=6]{5,-5,9,-9}{5}
\end{tikzpicture}
```

11.4 The Desargues graph with \grGeneralizedPetersen




```
\begin{tikzpicture}[rotate=90]
  \tikzstyle{VertexStyle} = [shape                = circle,%
                             color                = white,
                             fill                 = black,
                             very thin,
                             inner sep            = 0pt,%
                             minimum size         = 18pt,
                             draw]
  \tikzstyle{EdgeStyle}    = [thick,%
                              double              = brown,%
                              double distance     = 1pt]
  \grGeneralizedPetersen[Math,RA=6]{10}{3}
\end{tikzpicture}
```

SECTION 12

Doyle

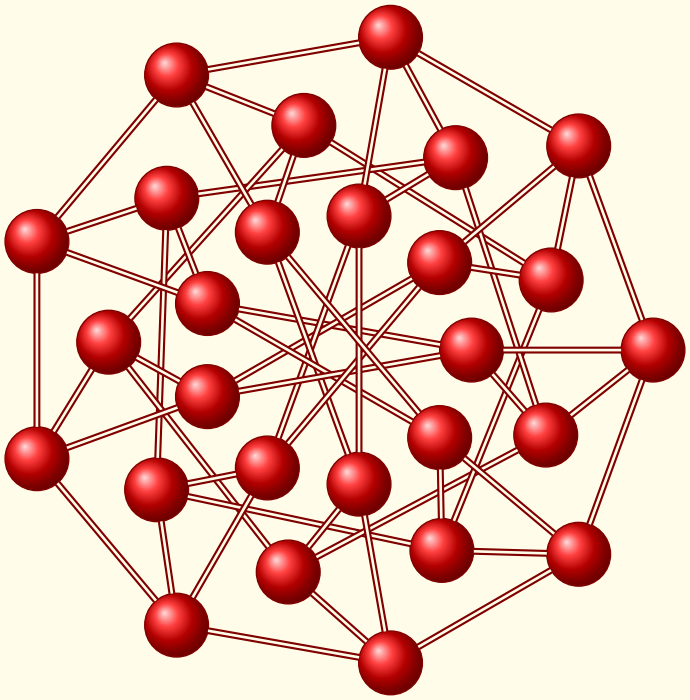
\grDoyle[⟨*options*⟩]

From MathWord : http://mathworld.wolfram.com/DoyleGraph.html

The Doyle graph, sometimes also known as the Holt graph (Marušič et al. 2005), is the symmetric quartic graph on 27 nodes illustrated. It is a Symmetric Graph. Three embeddings are illustrated below. MathWorld by E.Weisstein

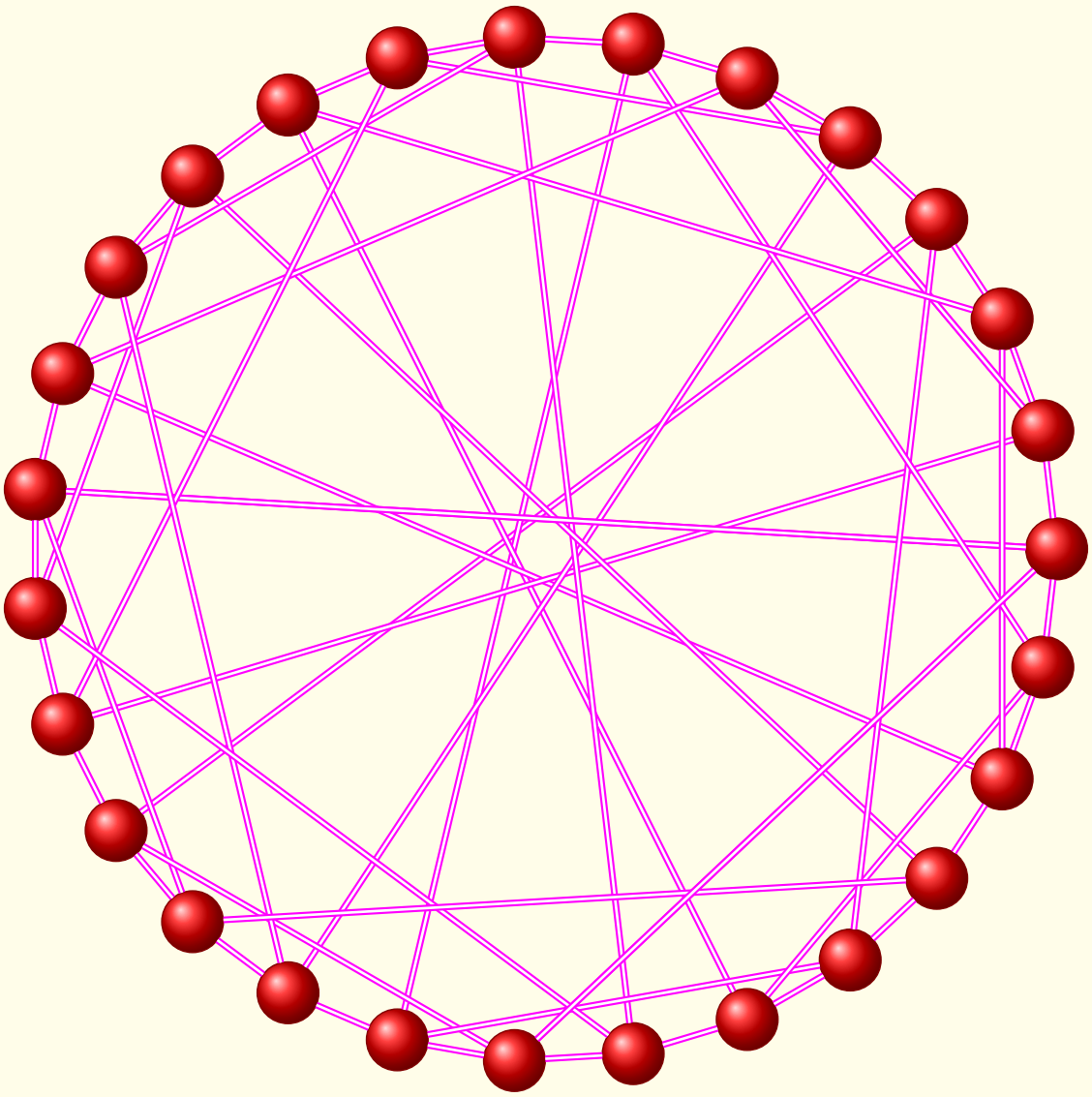
The Doyle graph is implemented in **tkz-berge** as **\grDoyle** with three forms.

12.1 The Doyle graph : form 1



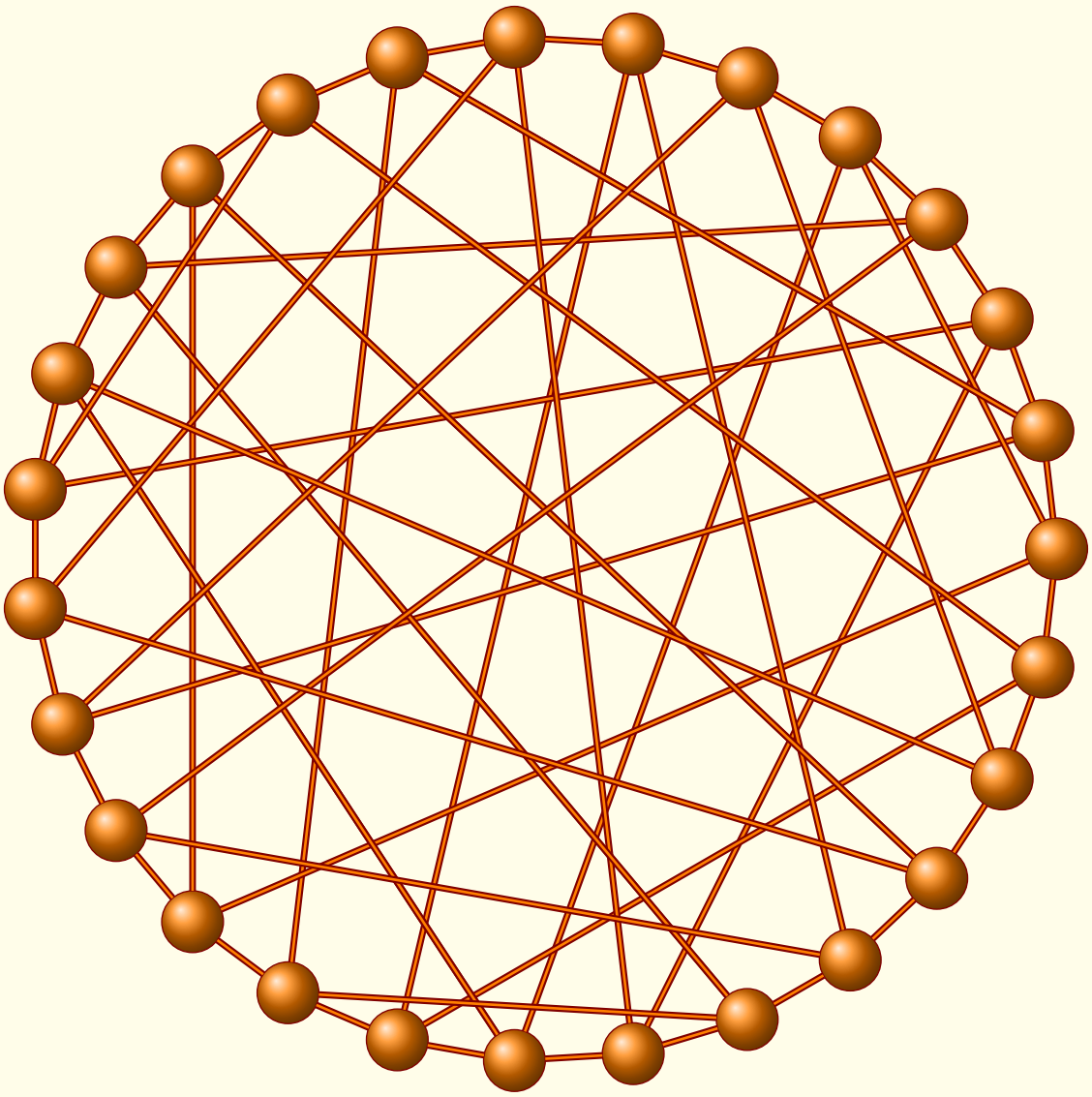
```
\begin{tikzpicture}[scale=.6]
  \GraphInit[vstyle=Shade]
  \SetGraphShadeColor{red}{Maroon}{fondpaille}
  \SetVertexNoLabel
  \grDoyle[RA=7,RB=5,RC=3]
\end{tikzpicture}
```

12.2 The Doyle graph : form 2



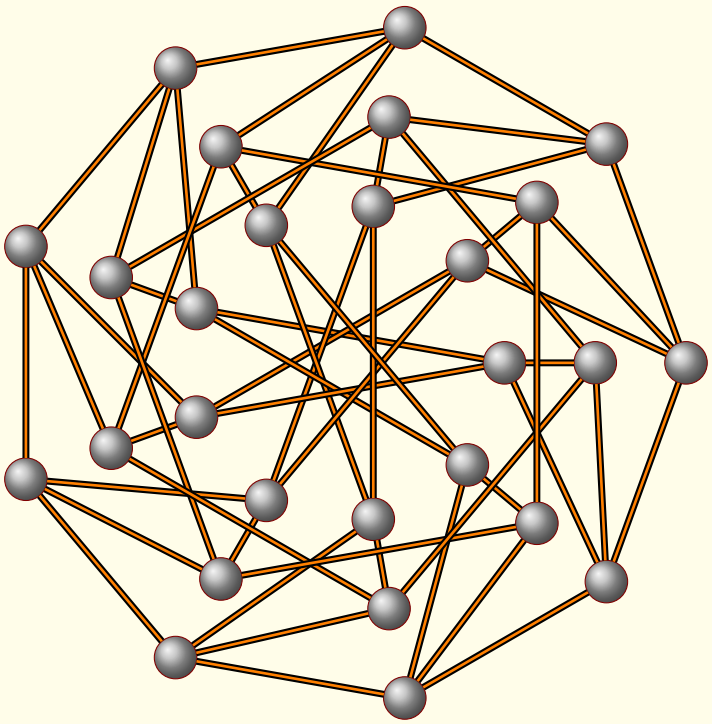
```
\begin{tikzpicture}
   \GraphInit[vstyle=Shade]
   \SetGraphShadeColor{red}{Magenta}{white}
   \SetVertexNoLabel
   \grDoyle[form=2,RA=7]
\end{tikzpicture}
```

12.3 The Doyle graph : form 3



```
\begin{tikzpicture}
  \SetGraphArtColor{red}{Magenta}{red}
   \GraphInit[vstyle=Shade]
   \SetVertexNoLabel
   \grDoyle[form=3,RA=7,RB=2]
\end{tikzpicture}
```

12.4 27 nodes but not isomorphic to the Doyle graph



```
\begin{tikzpicture}[scale=.6]
   \tikzstyle{VertexStyle} = [shape           = circle,
                              ball color      = gray!60,
                              minimum size    = 16pt,draw]
   \tikzstyle{EdgeStyle}   = [thick,color=black,%
                              double          = orange,%
                              double distance = 1pt]
   \SetVertexNoLabel
   \grCycle[RA=7.5]{9}
   \grEmptyCycle[prefix=b,RA=5.5]{9}
   \grCirculant[prefix=c,RA=3.5]{9}{4}
   \EdgeIdentity{b}{c}{9}
   \EdgeMod{a}{c}{9}{1}
   \EdgeMod{a}{b}{9}{1}
   \EdgeInGraphMod{b}{9}{2}
  \end{tikzpicture}
```

SECTION 13

Folkman

\grFolkman[⟨*options*⟩]

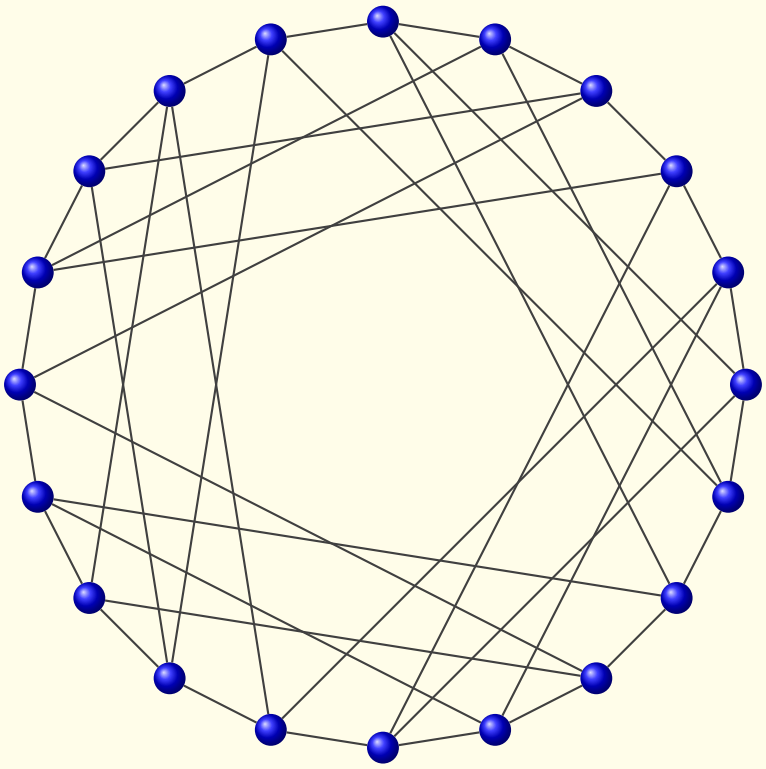
From MathWorld : http://mathworld.wolfram.com/FolkmanGraph.html

The Folkman graph is a semisymmetric graph that has the minimum possible number of nodes 20. MathWorld by E.Weisstein

13.1 Folkman Graph LCF embedding

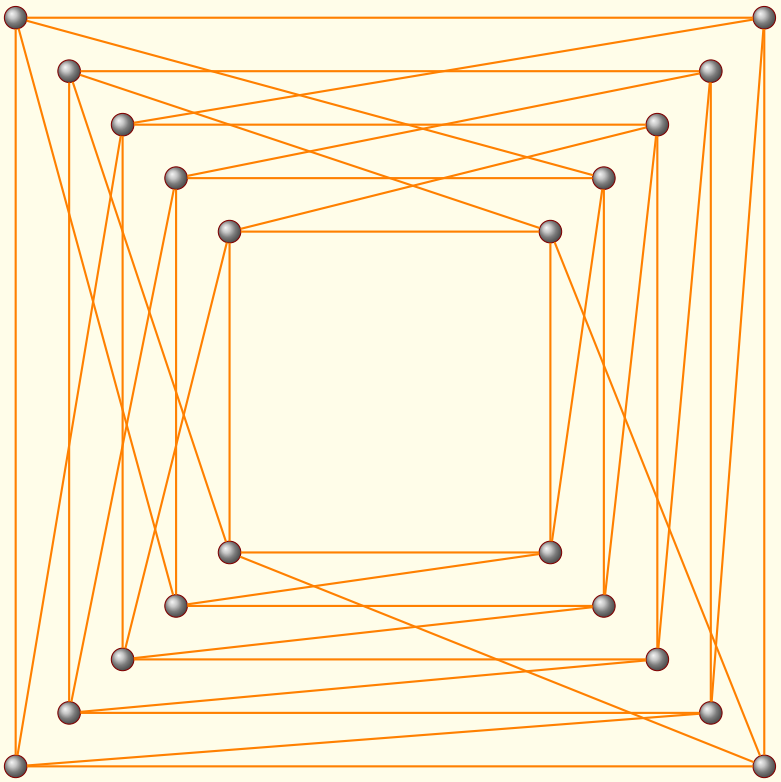
The code is

```
\grLCF[RA=7]{5,-7,-7,5}{5}
```



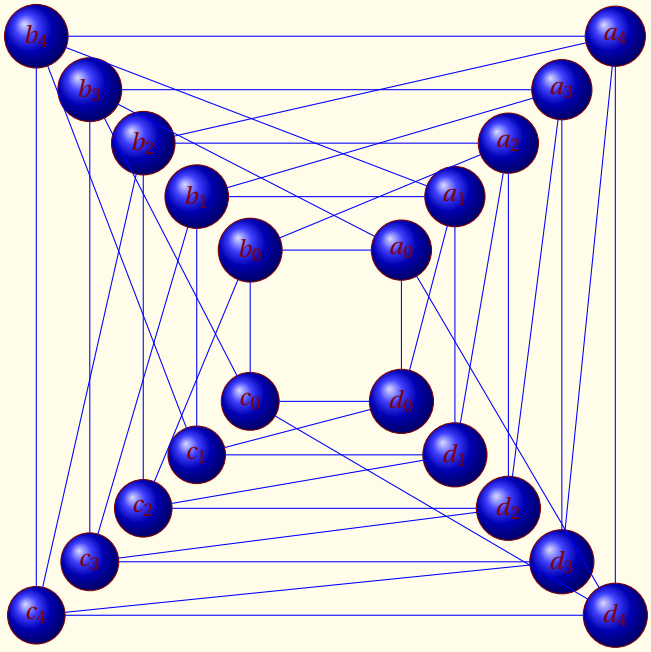
```
\begin{tikzpicture}[scale=.8]
      \GraphInit[vstyle=Art]
      \SetGraphArtColor{blue}{darkgray}
      \grFolkman[RA=6]
\end{tikzpicture}
```

13.2 Folkman Graph embedding 1



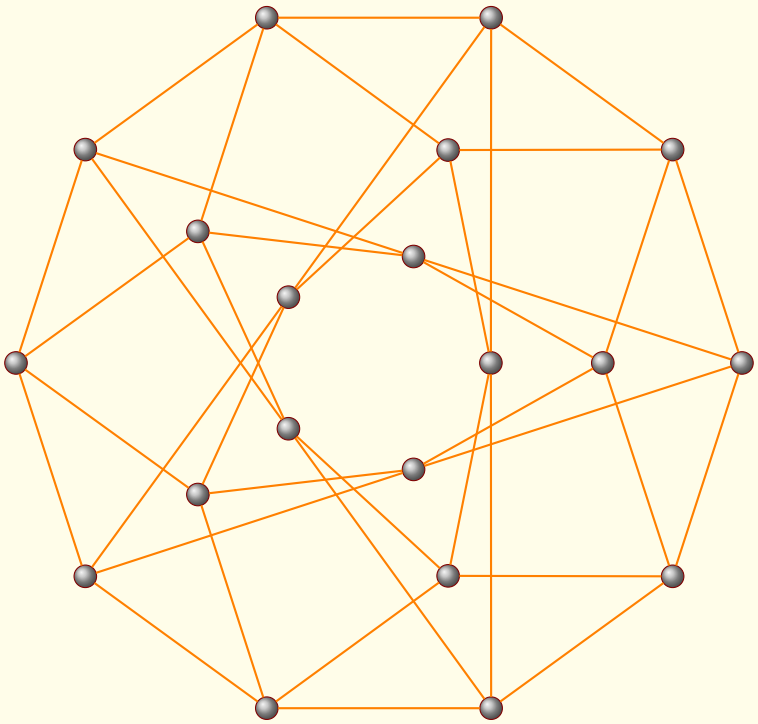
```
\begin{tikzpicture}[rotate=45]%
     \tikzstyle{VertexStyle} = [shape           = circle,
                                shading         = ball,
                                ball color      = gray!60,
                                inner sep       = 3pt,
                                draw]
     \tikzstyle{EdgeStyle}  = [thick,orange]
     \SetVertexNoLabel
     \grCycle[prefix=a,RA=3]{4}%
     \grCycle[prefix=b,RA=4]{4}%
     \grCycle[prefix=c,RA=5]{4}%
     \grCycle[prefix=d,RA=6]{4}%
     \grCycle[prefix=e,RA=7]{4}%
     \foreach \r/\s/\t in {a/d/e,b/e/a,c/a/b,d/b/c,e/c/d}{%
        \Edges(\r0,\s1,\r2,\t3,\r0)
        }
\end{tikzpicture}
```

13.3 Folkman Graph embedding 1 new code




```
\begin{tikzpicture}
\begin{scope}[shift={(1,1)},rotate=45]\grEmptyPath[prefix=a,RA=1]{5}
  \end{scope}
\begin{scope}[shift={(-1,1)},rotate=135]\grEmptyPath[prefix=b,RA=1]{5}
  \end{scope}
\begin{scope}[shift={(-1,-1)},rotate=225]\grEmptyPath[prefix=c,RA=1]{5}
  \end{scope}
\begin{scope}[shift={(1,-1)},rotate=315]\grEmptyPath[prefix=d,RA=1]{5}
  \end{scope}
  \EdgeIdentity*{a}{b}{0,...,4}  \EdgeIdentity*{b}{c}{0,...,4}
  \EdgeIdentity*{c}{d}{0,...,4}  \EdgeIdentity*{d}{a}{0,...,4}
  \EdgeDoubleMod{a}{5}{0}{1}{b}{5}{3}{1}{1}
  \EdgeDoubleMod{a}{5}{2}{1}{b}{5}{0}{1}{2}
  \EdgeDoubleMod{a}{5}{1}{1}{d}{5}{0}{1}{3}
  \EdgeDoubleMod{c}{5}{2}{1}{b}{5}{0}{1}{2}
  \EdgeDoubleMod{c}{5}{0}{1}{b}{5}{3}{1}{1}
  \EdgeDoubleMod{c}{5}{1}{1}{d}{5}{0}{1}{3}
  \Edges(a0,d4,c0)
\end{tikzpicture}
```

13.4 Folkman Graph embedding 3



```
\begin{tikzpicture}[scale=.8]
   \SetVertexNoLabel
   \tikzstyle{VertexStyle} = [shape           = circle,
                              shading         = ball,
                              ball color      = gray!60,
                              inner sep       = 3pt,
                              draw]
   \tikzstyle{EdgeStyle}    = [thick,orange]
   \grEmptyCycle[prefix=a,RA=1.85]{5} \grEmptyCycle[prefix=b,RA=3.7]{5}
   \grCycle[prefix=c,RA=6]{10}
   \EdgeDoubleMod{a}{5}{0}{1}{b}{5}{1}{1}{4}
   \EdgeDoubleMod{a}{5}{0}{1}{b}{5}{4}{1}{4}
   \EdgeDoubleMod{b}{5}{0}{1}{c}{10}{9}{2}{4}
   \EdgeDoubleMod{b}{5}{0}{1}{c}{10}{1}{2}{4}
   \EdgeDoubleMod{a}{5}{0}{1}{c}{10}{8}{2}{4}
   \EdgeDoubleMod{a}{5}{0}{1}{c}{10}{2}{2}{4}
\end{tikzpicture}
```

SECTION 14

Foster

\grFoster[⟨*options*⟩]

From MathWord : http://mathworld.wolfram.com/FosterGraph.html

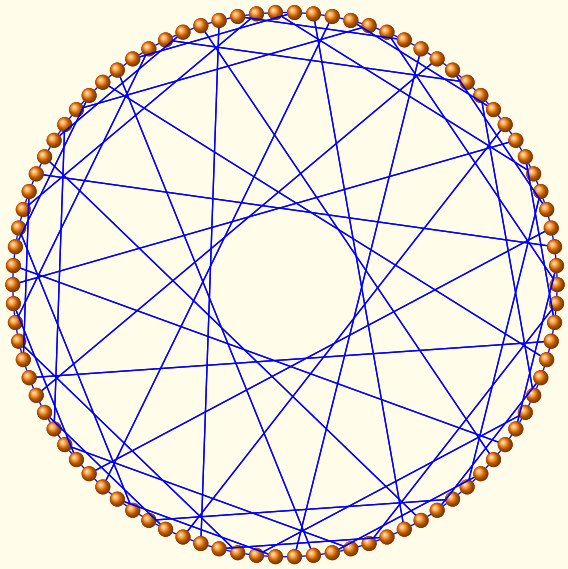
The Foster graph is a graph on 90 vertices and 135 arcs. It has a unique order-15 LCF notations.

MathWorld by E.Weisstein

14.1 Foster graph

The macros is based on

```
\grLCF[Math,RA=7]{17, -9, 37, -37, 9, -17}{15}
```



```
\begin{tikzpicture}[scale=.6]
  \renewcommand*{\VertexInnerSep}{2pt}
  \renewcommand*{\EdgeLineWidth}{0.5pt}
  \GraphInit[vstyle=Art]
  \tikzset{VertexStyle/.append style={minimum size=2pt}}
  \SetGraphColor{red}{blue}
  \grLCF[Math,RA=6]{17, -9, 37, -37, 9, -17}{15}
\end{tikzpicture}
```

SECTION 15

Franklin

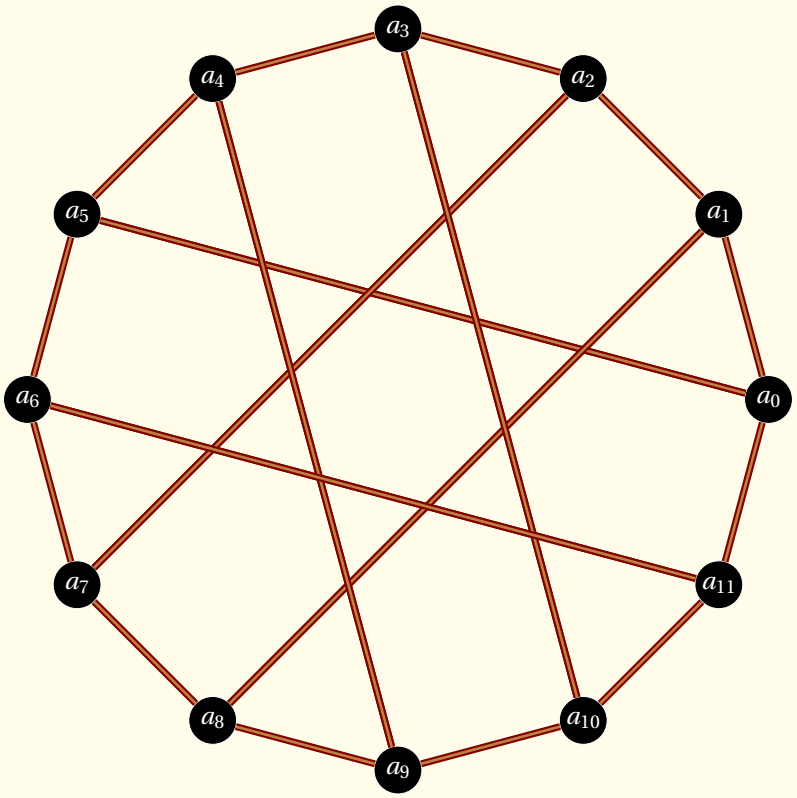
`\grFranklin[⟨options⟩]`

From MathWord : http://mathworld.wolfram.com/FranklinGraph.html

The Franklin graph is the 12-vertex cubic graph shown above whose embedding on the Klein bottle divides it into regions having a minimal coloring using six colors, thus providing the sole counterexample to the Heawood conjecture. MathWorld by E.Weisstein

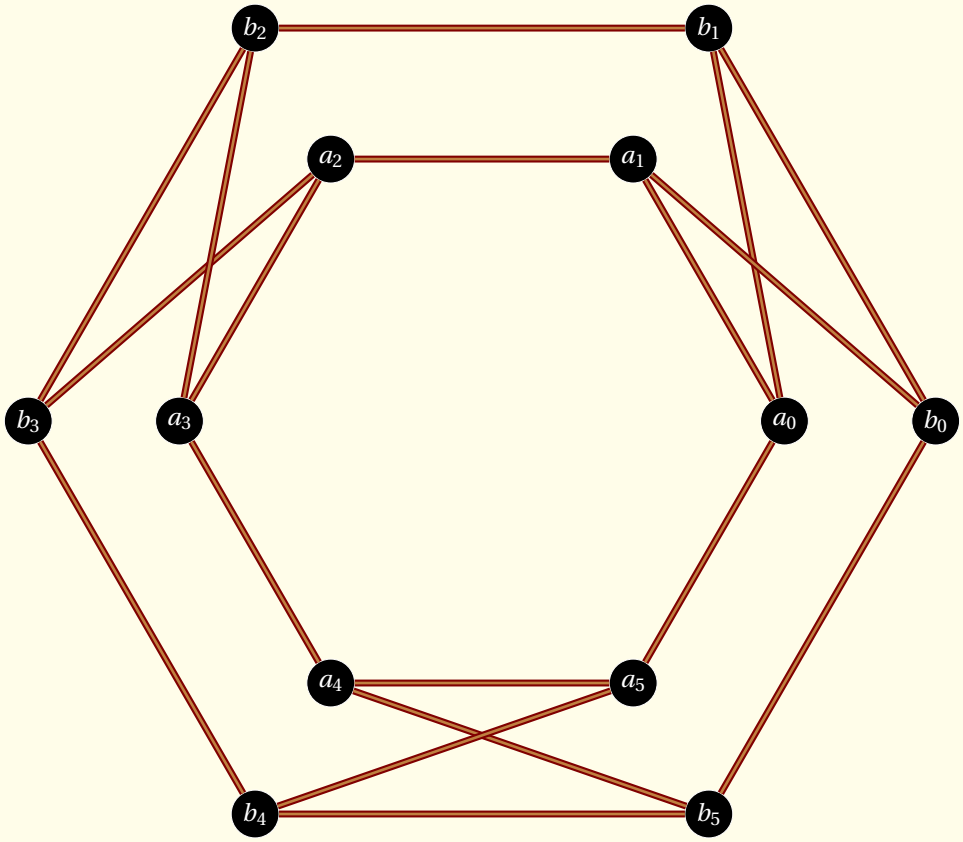
The Franklin graph is implemented in **tkz-berge** as **\grFranklin**.

15.1 The Franklin graph : embedding 1




```
\begin{tikzpicture}[scale=.7]
   \grFranklin[Math,RA=7]
\end{tikzpicture}
```

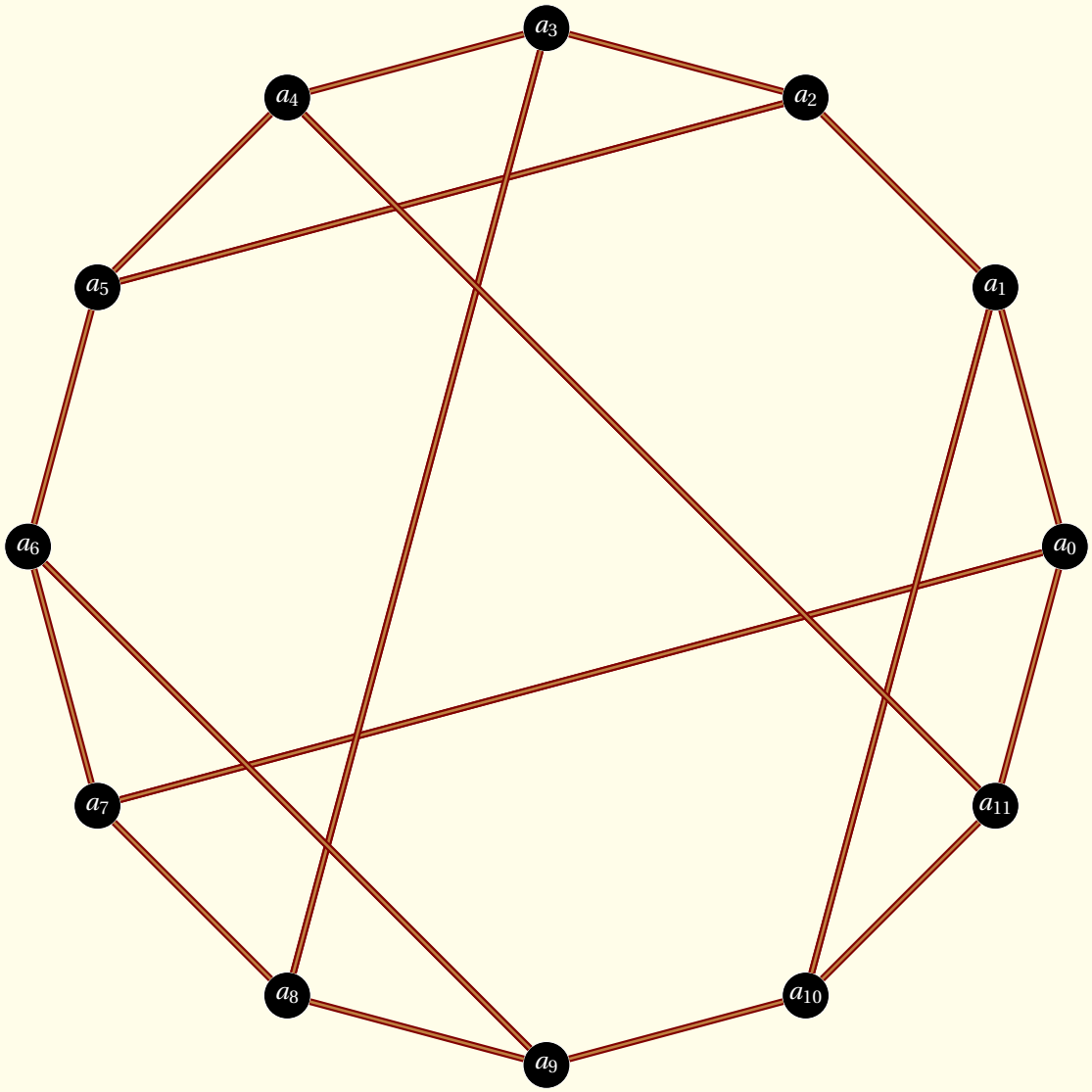
15.2 The Franklin graph : embedding 2




```
\begin{tikzpicture}
  \grCycle[Math,RA=4,prefix=a]{6}
  \grCycle[Math,RA=6,prefix=b]{6}
  \foreach \x in {0,...,5}{%
    \ifthenelse{\isodd{\x}}{%
     \pgfmathsetcounter{tempi}{\x-1}}{%
     \pgfmathsetcounter{tempi}{\x+1}}
     \Edge(a\x)(b\thetempi)
}
\end{tikzpicture}
```

15.3 The Franklin graph : with LCF notation embedding 3

*2cm




```
\begin{tikzpicture}
   \grLCF[Math,RA=7]{-5,-3,3,5}{3}
\end{tikzpicture}
```

SECTION 16

Gray

From MathWorld :http://mathworld.wolfram.com/GrayGraph.html

MathWorld by E.Weisstein

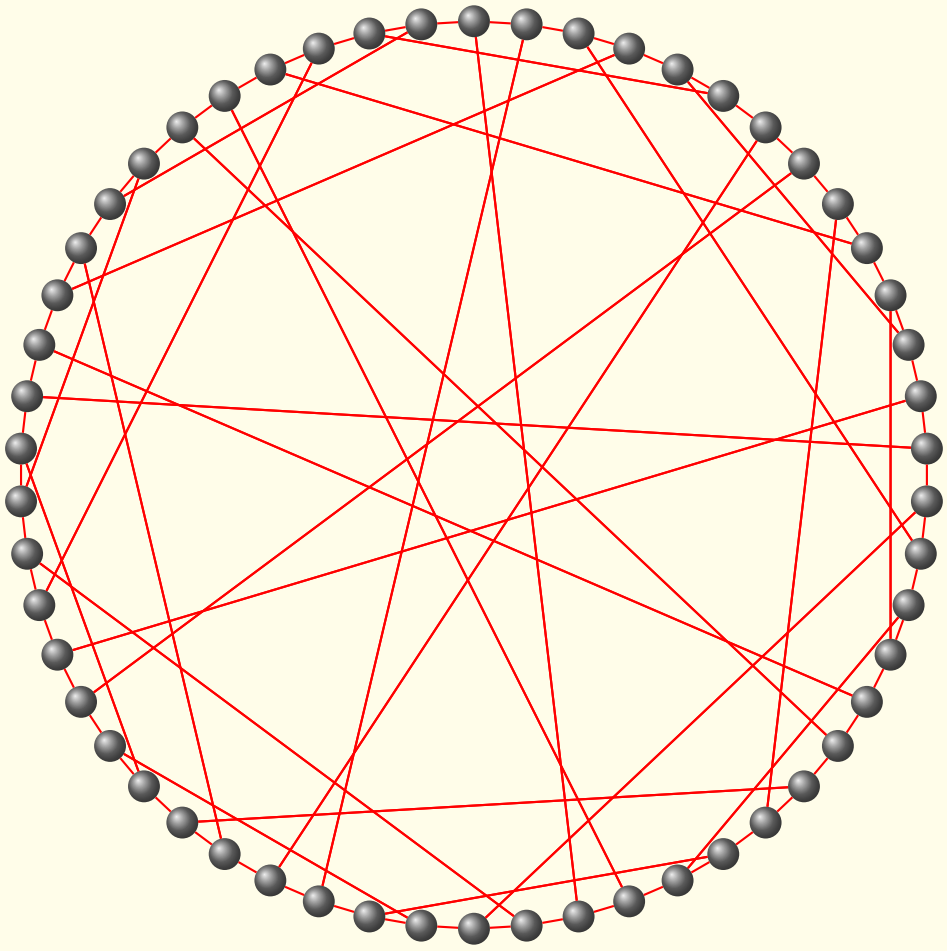
The Gray graph is a cubic semisymmetric graph on 54 vertices. It was discovered by Marion C. Gray in 1932, and was first published by Bouwer (1968). Malnic et al. (2004) showed that the Gray graph is indeed the smallest possible cubic semisymmetric graph.

It is the incidence graph of the Gray configuration.

The Gray graph has a single order-9 LCF Notation and five distinct order-1 LCF notations.

The Gray graph has girth 8, graph diameter 6

It can be represented in LCF notation as $[-25,7,-7,13,-13,25]^9$



```
\begin{tikzpicture}[rotate=90]
    \GraphInit[vstyle=Art]
    \SetGraphArtColor{gray}{red}
    \grLCF[Math,RA=6]{-25,7,-7,13,-13,25}{9}
\end{tikzpicture}
```

SECTION 17

Groetzsch

\grGrotzsch[⟨*options*⟩]{⟨*k*⟩}

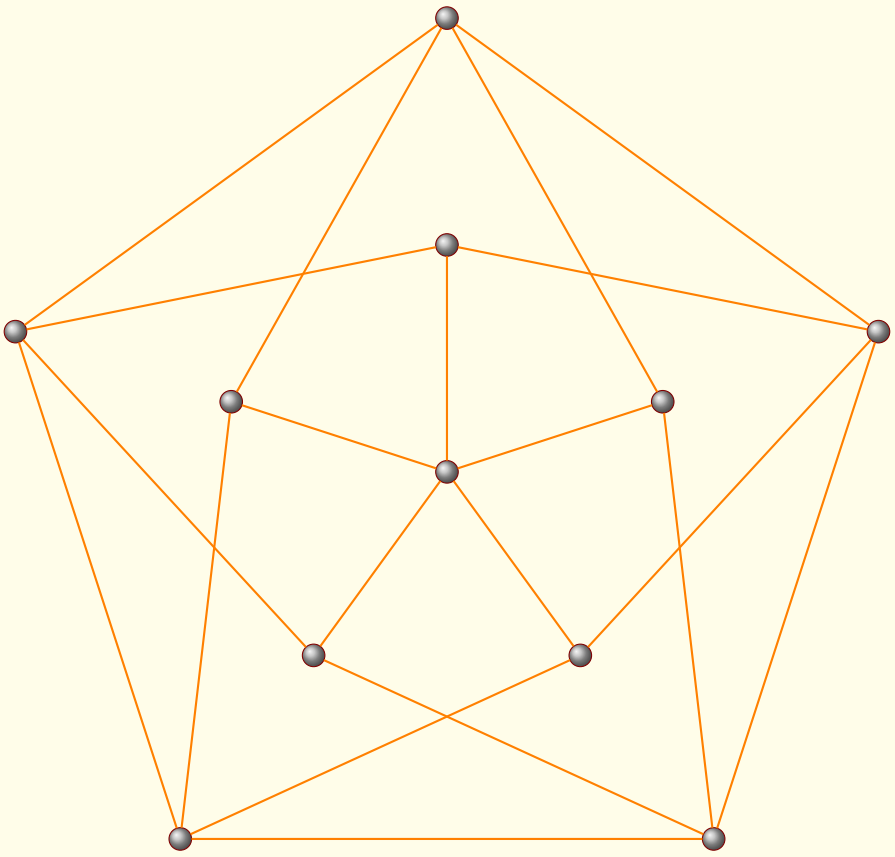
From Wikipedia : http://en.wikipedia.org/wiki/Grötzsch_graph

The Grötzsch graph is a triangle-free graph with 11 vertices, 20 edges, and chromatic number 4. It is named after German mathematician Herbert Grötzsch, and its existence demonstrates that the assumption of planarity is necessary in Grötzsch's theorem (Grötzsch 1959) that every triangle-free planar graph is 3-colorable.

From MathWord : http://mathworld.wolfram.com/GroetzschGraph.html

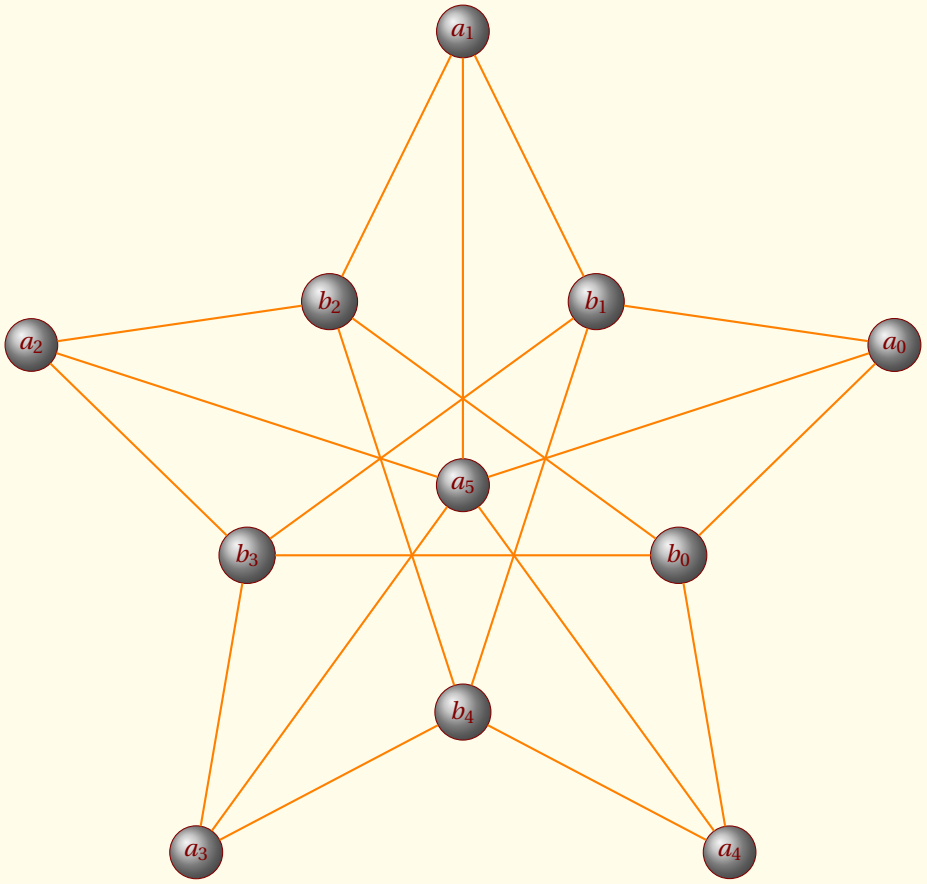
The Grötzsch graph is smallest triangle-free graph with chromatic number four. It is identical to the Mycielski Graph of order four. MathWorld by E.Weisstein

17.1 Grotzsch Graph : first form



```
  \begin{tikzpicture}
   \grGrotzsch[RA=3,RB=6]{6}%
\end{tikzpicture}
```

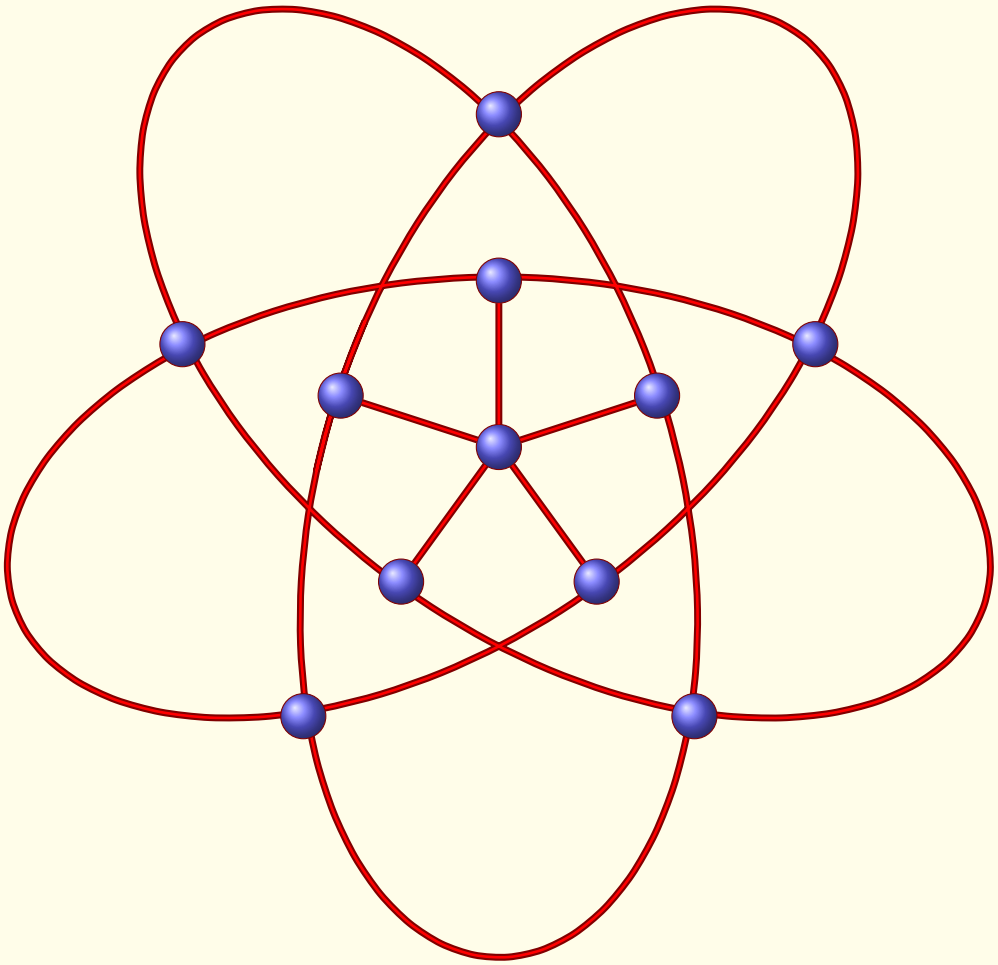
17.2 Grotzsch Graph : second form




```
\begin{tikzpicture}
   \grGrotzsch[form=2,RA=6,RB=3]{6}%
\end{tikzpicture}
```

17.3 Grotzsch Graph : third form

From Wikipedia : http://en.wikipedia.org/wiki/Complete_bipartite_graph



```
\begin{tikzpicture}[rotate=-18]
 \draw[scale=.5,samples at={-6.4,-6.3,...,6.4},
           smooth,thick,
           variable=\t,
           double= red,
           double distance = 1pt]
      plot ({3*(1.5*cos(\t r) +3*cos(1.5*\t r))},%
            {3*(1.5*sin(\t r) -3*sin(1.5*\t r))});
\begin{scope}[rotate=36]
   \grStar[prefix=a,RA=2.2]{6}%
   \grEmptyCycle[prefix=b,RA=4.4]{5}%
\end{scope}
\end{tikzpicture}
```

SECTION 18

Heawood graph

`\grHeawood[⟨options⟩]`

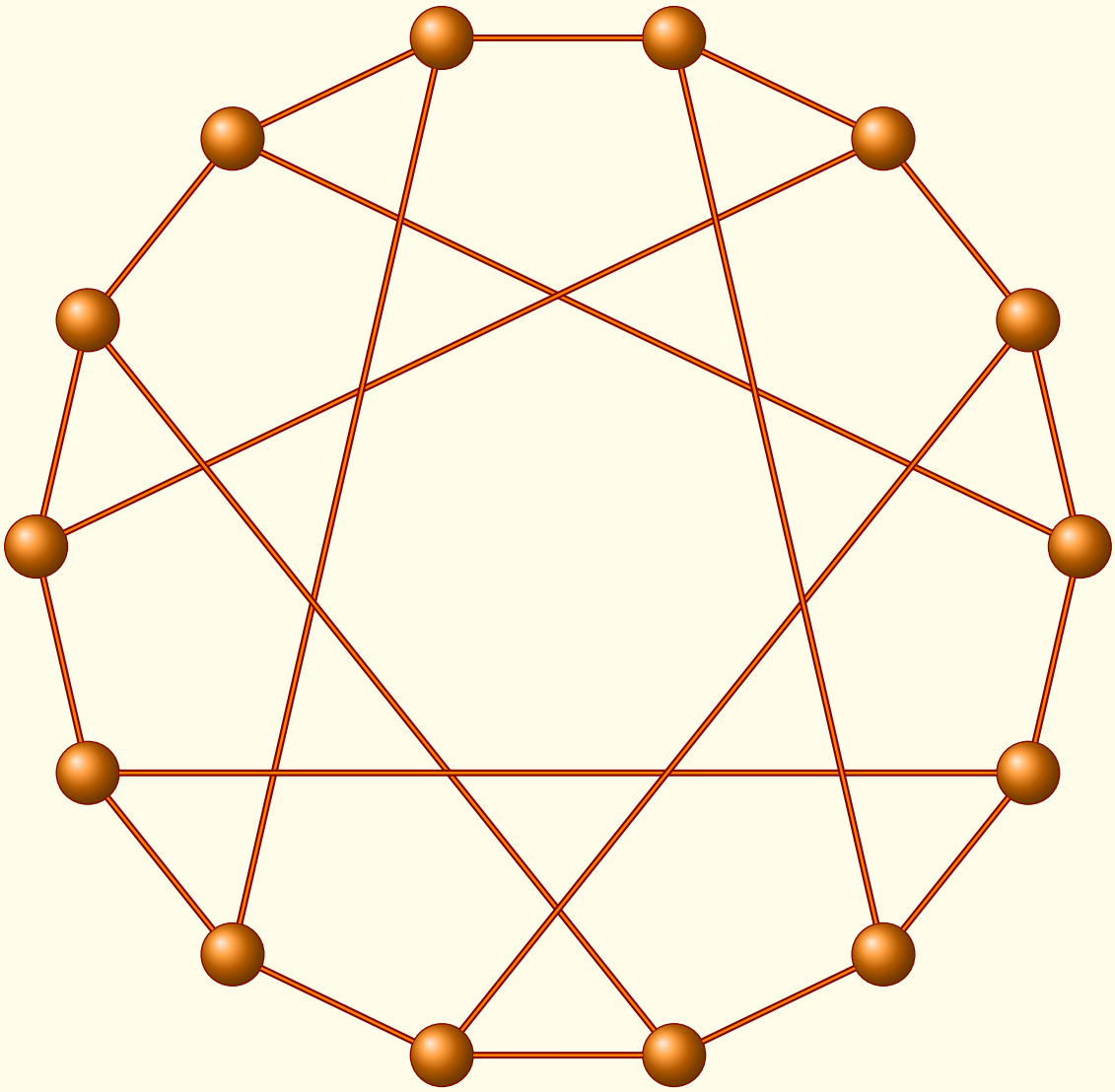
From Wikipedia http://en.wikipedia.org/wiki/Heawood_graph

The Heawood graph is an undirected graph with 14 vertices and 21 edges. Each vertex is adjacent to exactly three edges (that is, it is a cubic graph), and all cycles in the graph have six or more edges. Percy John Heawood (1861-1955) was an English mathematician who spent a large amount of time on questions related to the four colour theorem.

From MathWorld http://mathworld.wolfram.com/HeawoodGraph.html

The Heawood graph is the unique $(3,6)$*-cage graph and Moore graph and is graph illustrated below in one of his embeddings.* MathWorld by E.Weisstein

18.1 Heawood graph

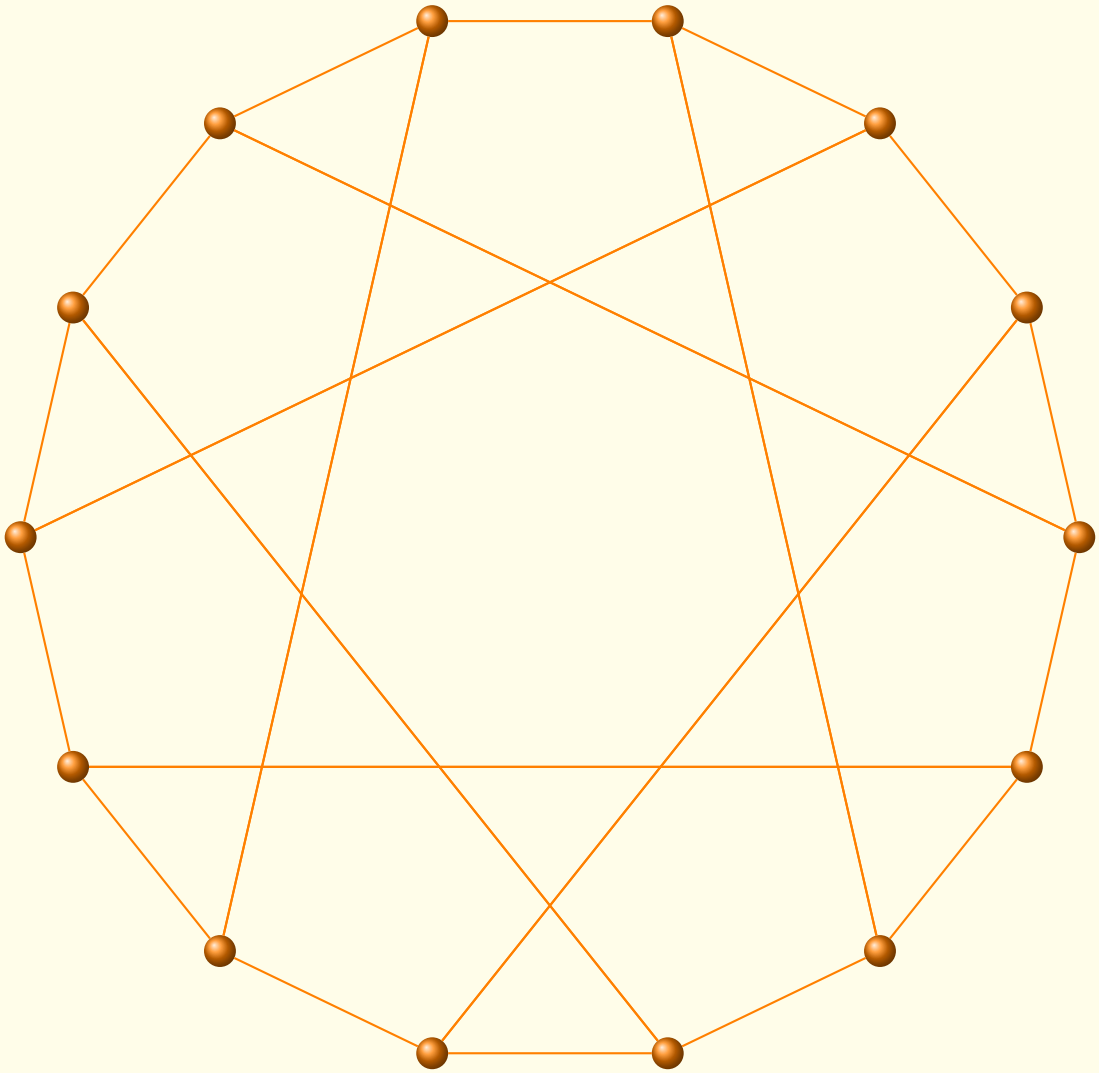


```
\begin{tikzpicture}%
   \GraphInit[vstyle=Shade]
   \grHeawood[RA=7]
\end{tikzpicture}
```

It can be represented in LCF notation as $[5,-5]^7$

\grLCF[RA=5]{5,9}{7} gives the result because $-5 = 9 \ mod \ 14$.

18.2 Heawood graph with LCF notation



```
\begin{tikzpicture}%
   \GraphInit[vstyle=Art]
   \grLCF[RA=7]{5,9}{7}%
\end{tikzpicture}
```

SECTION 19

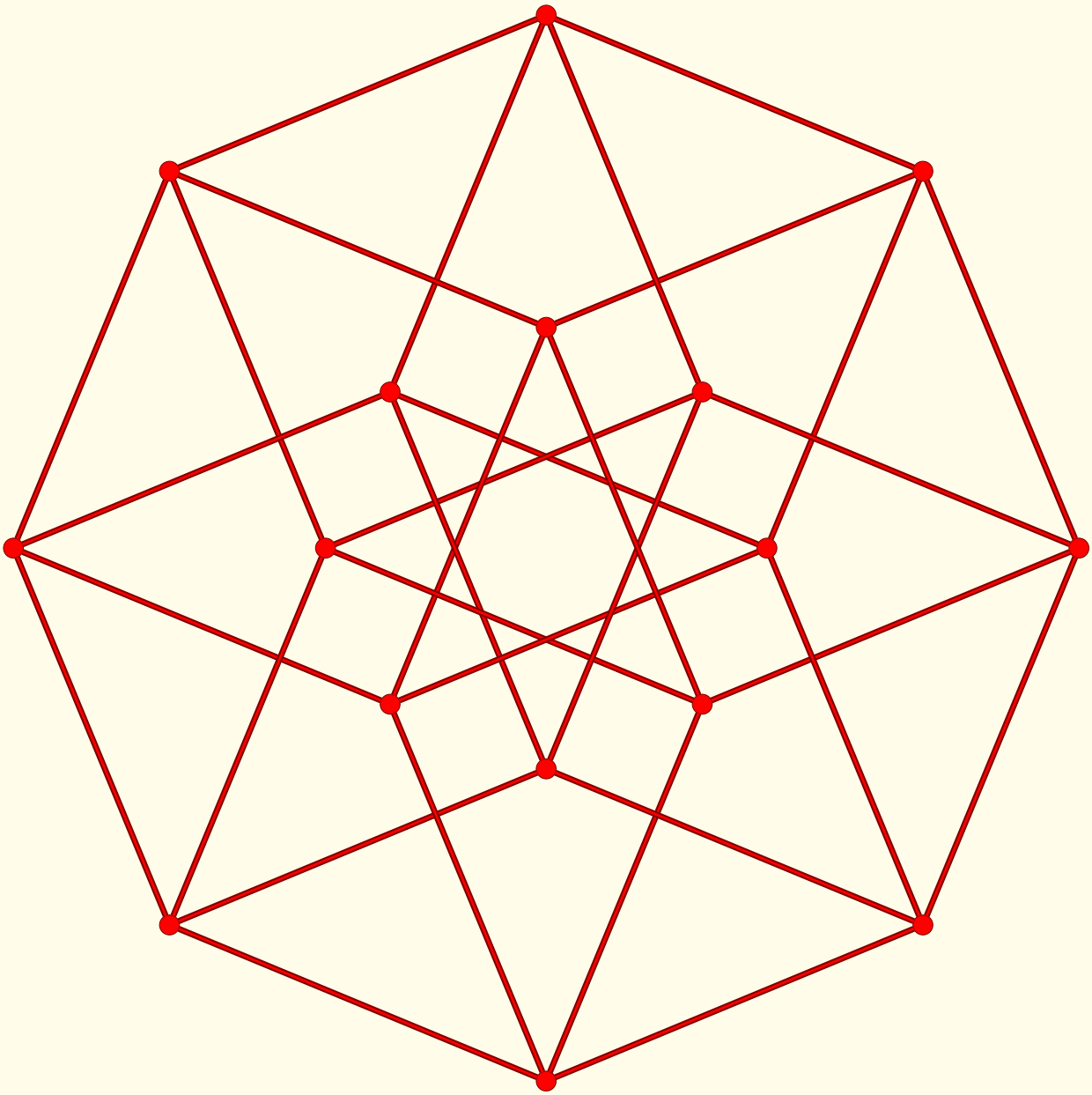
Hypercube

From Wikipedia :http://en.wikipedia.org/wiki/Hypercube_graph

In the mathematical field of graph theory, the hypercube graph Q_n is a special regular graph with $2n$ vertices, which correspond to the subsets of a set with n elements. Two vertices labelled by subsets S and T are joined by an edge if and only if S can be obtained from T by adding or removing a single element. Each vertex of Q_n is incident to exactly n edges (that is, Q_n is n-regular), so the total number of edges is $2^{n-1}n$. The name comes from the fact that the hypercube graph is the one-dimensional skeleton of the geometric hypercube. Hypercube graphs should not be confused with cubic graphs, which are graphs that are 3-regular. The only hypercube that is a cubic graph is Q_3.

19.1 The hypercube graph Q_4

The code is on the next page.



```
\begin{tikzpicture}
  \grCycle[RA=8]{8}
  \pgfmathparse{8*(1-4*sin(22.5)*sin(22.5))}
  \let\tkzbradius\pgfmathresult
  \grCirculant[prefix=b,RA=\tkzbradius]{8}{3}
  \makeatletter
  \foreach \vx in {0,...,7}{%
    \pgfmathsetcounter{tkz@gr@n}{mod(\vx+1,8)}
    \pgfmathsetcounter{tkz@gr@a}{mod(\vx+7,8)}
    \pgfmathsetcounter{tkz@gr@b}{mod(\thetkz@gr@n+1,8)}
    \Edge(a\thetkz@gr@n)(b\thetkz@gr@b)
    \Edge(b\thetkz@gr@a)(a\vx)
    }
  \makeatother
\end{tikzpicture}
```

SECTION 20

The Seven Bridges of Königsberg

`\grKonisberg[⟨options⟩]{⟨k⟩}`

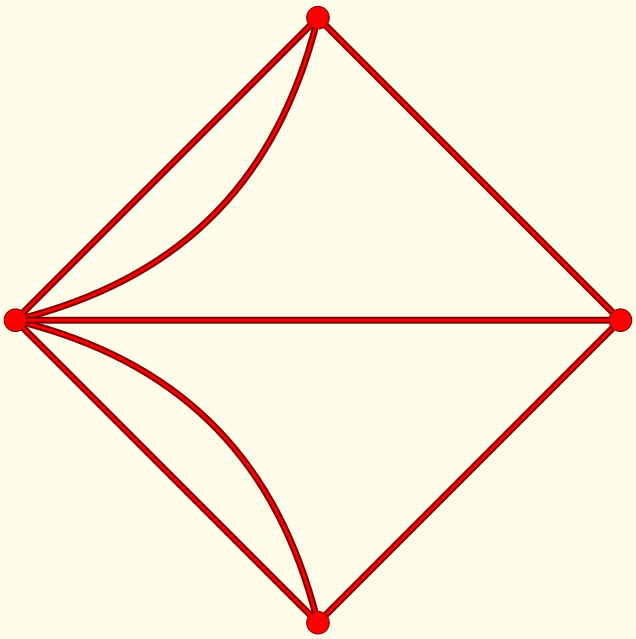
From MathWorld : http://mathworld.wolfram.com/KoenigsbergBridgeProblem.html

The Königsberg bridge problem asks if the seven bridges of the city of Königsberg (left figure; Kraitchik 1942), formerly in Germany but now known as Kaliningrad and part of Russia, over the river Preger can all be traversed in a single trip without doubling back, with the additional requirement that the trip ends in the same place it began. This is equivalent to asking if the multigraph on four nodes and seven edges (right figure) has an Eulerian circuit. This problem was answered in the negative by Euler (1736), and represented the beginning of graph theory. MathWorld by E.Weisstein

From Wikipedia : http://en.wikipedia.org/wiki/Seven_Bridges_of_Königsberg

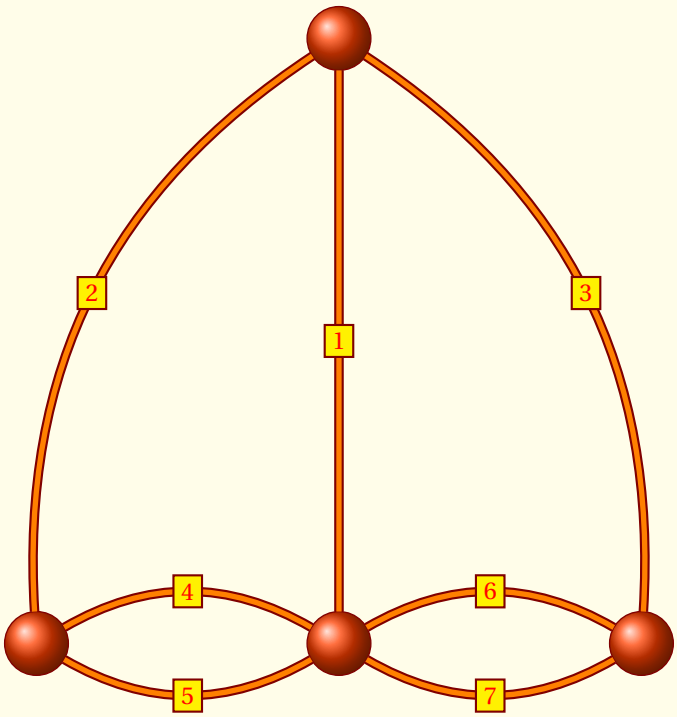
The paper written by Leonhard Euler on the Seven Bridges of Königsberg and published in 1736 is regarded as the first paper in the history of graph theory.
The Seven Bridges of Königsberg is a famous solved mathematics problem inspired by an actual place and situation. The city of Königsberg, Prussia (now Kaliningrad, Russia) is set on the Pregel River, and included two large islands which were connected to each other and the mainland by seven bridges. The problem is to decide whether it is possible to walk a route that crosses each bridge exactly once.
In 1736, Leonhard Euler proved that it was not possible. In proving the result, Euler formulated the problem in terms of graph theory, by abstracting the case of Königsberg — first, by eliminating all features except the landmasses and the bridges connecting them; second, by replacing each landmass with a dot, called a vertex or node, and each bridge with a line, called an edge or link. The resulting mathematical structure is called a graph.

20.1 Königsberg graph with \grKonisberg



```
 \begin{tikzpicture}[node distance=4cm]
  \grKonisberg
\end{tikzpicture}
```

20.2 \Königsberg graph : fine embedding




```
\begin{tikzpicture}
  \renewcommand*{\VertexBallColor}{orange!50!red}
  \renewcommand*{\EdgeDoubleDistance}{2pt}
  \SetGraphUnit{4}
  \GraphInit[vstyle=Shade]
  \tikzset{LabelStyle/.style =   {draw,
                                  fill  = yellow,
                                  text  = red}}
  \Vertex{A}
  \EA(A){B}
  \EA(B){C}
  {\SetGraphUnit{8}
  \NO(B){D}}
  \Edge[label=1](B)(D)
  \tikzset{EdgeStyle/.append style = {bend left}}
  \Edge[label=4](A)(B)
  \Edge[label=5](B)(A)
  \Edge[label=6](B)(C)
  \Edge[label=7](C)(B)
  \Edge[label=2](A)(D)
  \Edge[label=3](D)(C)
\end{tikzpicture}
```

SECTION 21

Levi Graph

\grLevi[⟨*options*⟩]

From Wikipedia http://en.wikipedia.org/wiki/Levi_graph

In combinatorics a Levi graph or incidence graph is a bipartite graph associated with an incidence structure. From a collection of points and lines in an incidence geometry or a projective configuration, we form a graph with one vertex per point, one vertex per line, and an edge for every incidence between a point and a line.
In the mathematical field of graph theory, the Tutte–Coxeter graph or Tutte eight-cage is a 3-regular graph with 30 vertices and 45 edges. As the unique smallest cubic graph of girth 8 it is a cage and a Moore graph. It is bipartite, and can be constructed as the Levi graph of the generalized quadrangle.

From MathWord : http://mathworld.wolfram.com/LeviGraph.html

It has 30 nodes and 45 edges. It has girth 8, diameter 4, chromatic number 2. The Levi graph is a generalized polygon which is the point/line incidence graph of the generalized quadrangle . The graph was first discovered by Tutte (1947), and is also called the Tutte-Coxeter graph , Tutte's cage or "Tutte's (3,8)-cage".The Levi graph is the unique (3,8)-cage graph.
The incidence graph of a generic configuration is sometimes known as a Levi graph (Coxeter 1950).

MathWorld by E.Weisstein

Some examples of Levi Graphs with this definition are :

- Desargues graph
- Heawood graph
- Heawood graph
- Pappus graph
- Gray graph
- Tutte eight-cage

The two forms can be draw with :

```
\grLevi[RA=7]
```

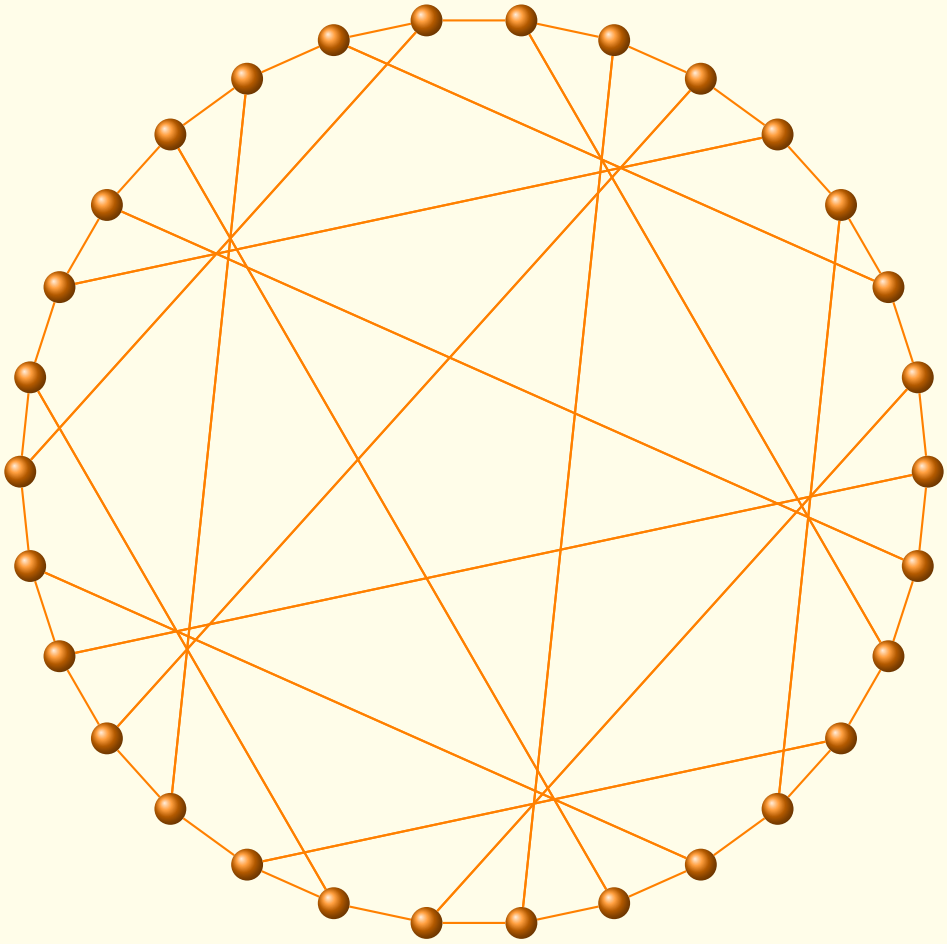
and

```
\grLevi[form=2,RA=7,RB=5,RC=3]
```

You can see on the next pages, the two forms.

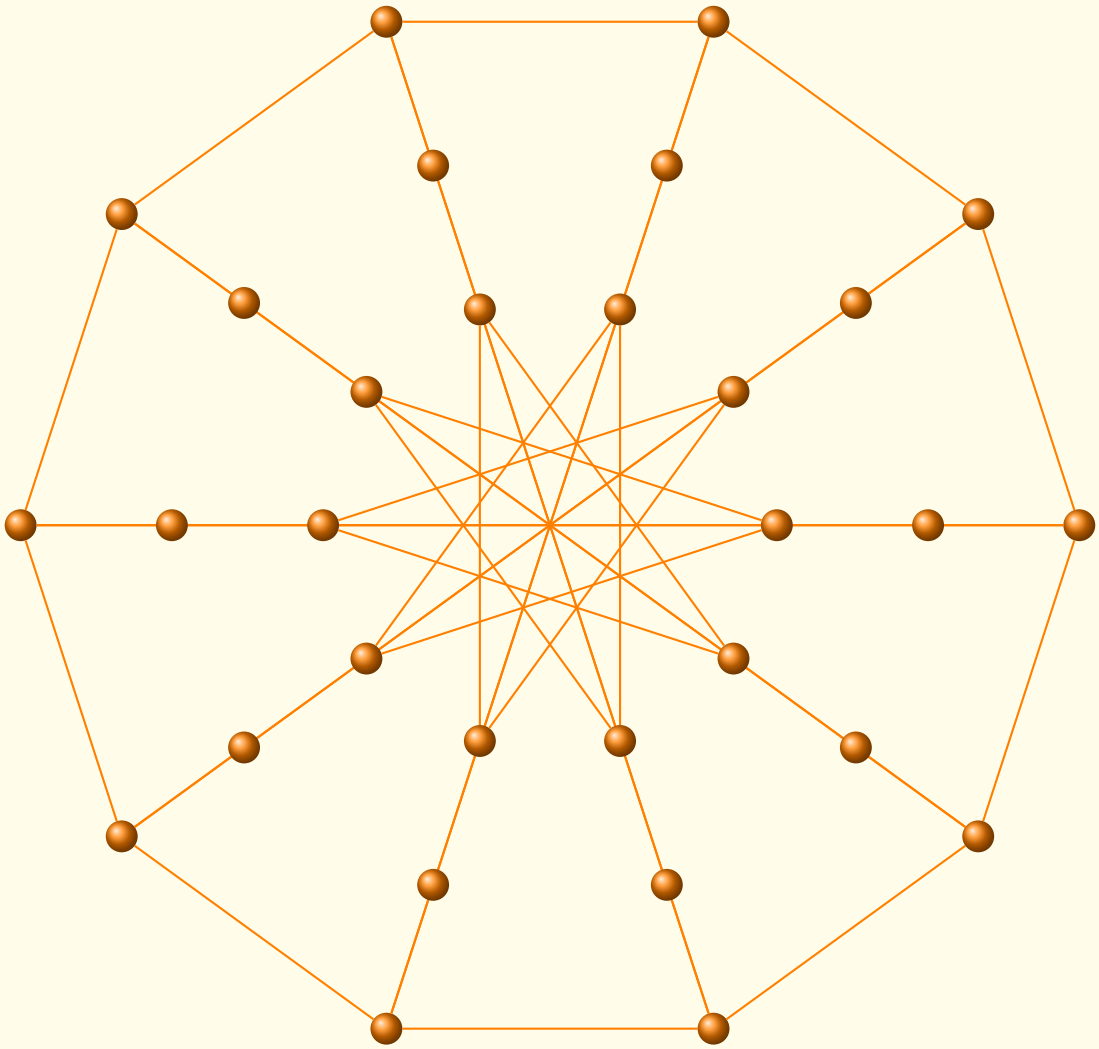
Now I show you how to code this graph.

21.1 Levy graph :form 1



```
  \begin{tikzpicture}
    \GraphInit[vstyle=Art]
    \grLCF[prefix=a,RA=6]{-13,-9,7,-7,9,13}{5}
\end{tikzpicture}
```

21.2 Levy graph :form 2



```
\begin{tikzpicture}
  \GraphInit[vstyle=Art]
  \grCycle[prefix=a,RA=7]{10}
  \EdgeInGraphMod{a}{10}{5}
  \grEmptyCycle[prefix=b,RA=5]{10}
  \grEmptyCycle[prefix=c,RA=3]{10}
  \EdgeInGraphMod{c}{10}{4}
\end{tikzpicture}
```

SECTION 22

Mc Gee

\grMcGee[⟨*options*⟩]

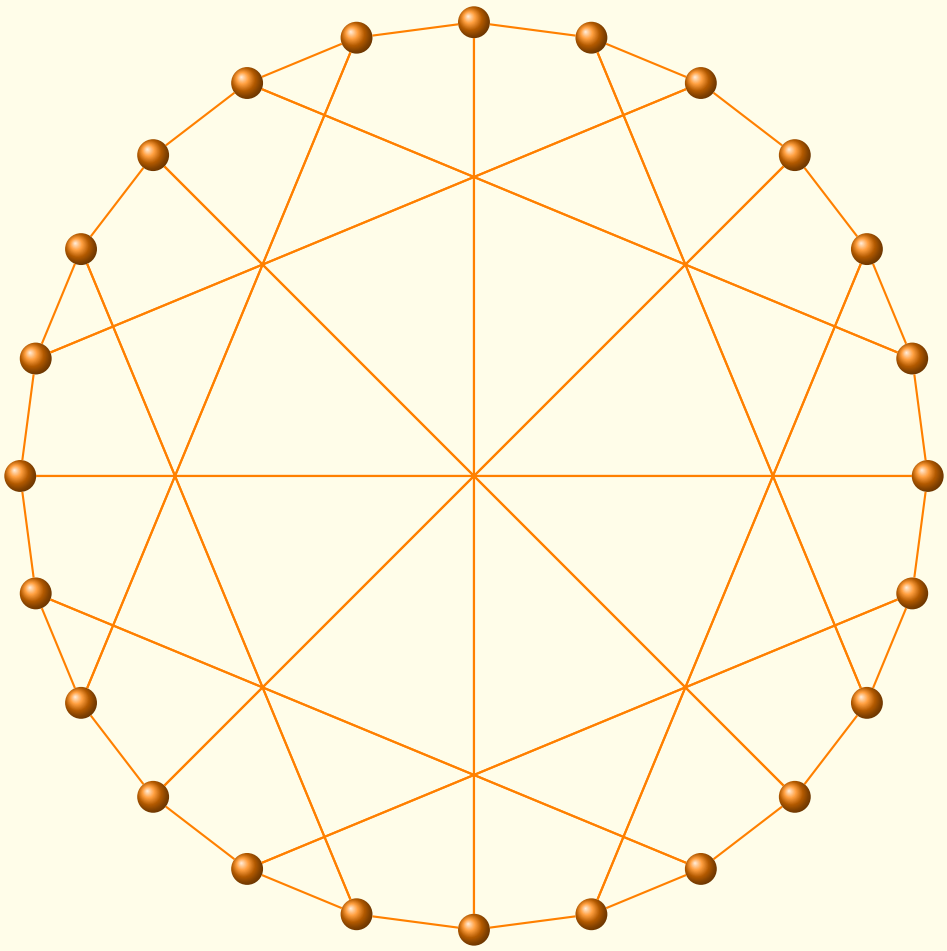
From MathWord : http://mathworld.wolfram.com/McGeeGraph.html

The McGee graph is the unique 7-cage graph. It has 24 nodes, 36 edges, girth 7, diameter 4, and is a cubic graph. It has chromatic number 3. MathWorld by E.Weisstein

22.1 McGee graph with \grMcGee

The same result is obtained with

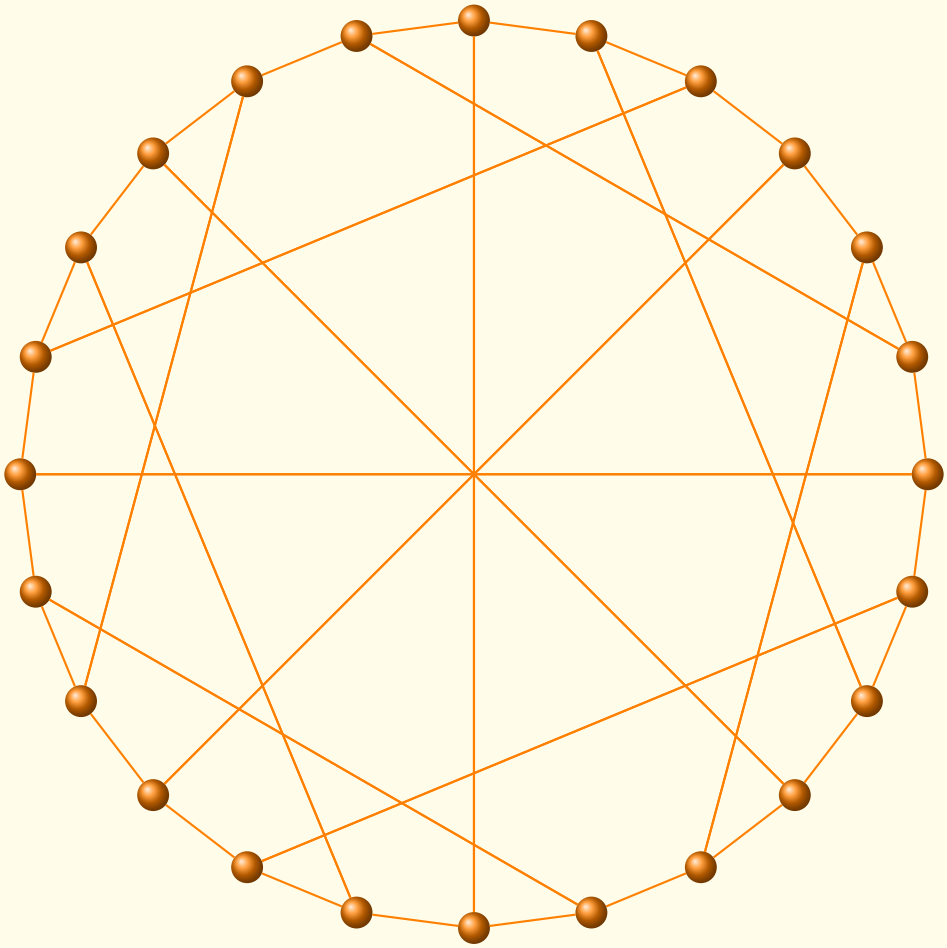
```
\grLCF[Math,RA=6]{-12,7,-7}{8}
```



```
\begin{tikzpicture}[rotate=90]
   \GraphInit[vstyle=Art]
    \grMcGee[Math,RA=6]
\end{tikzpicture}
```

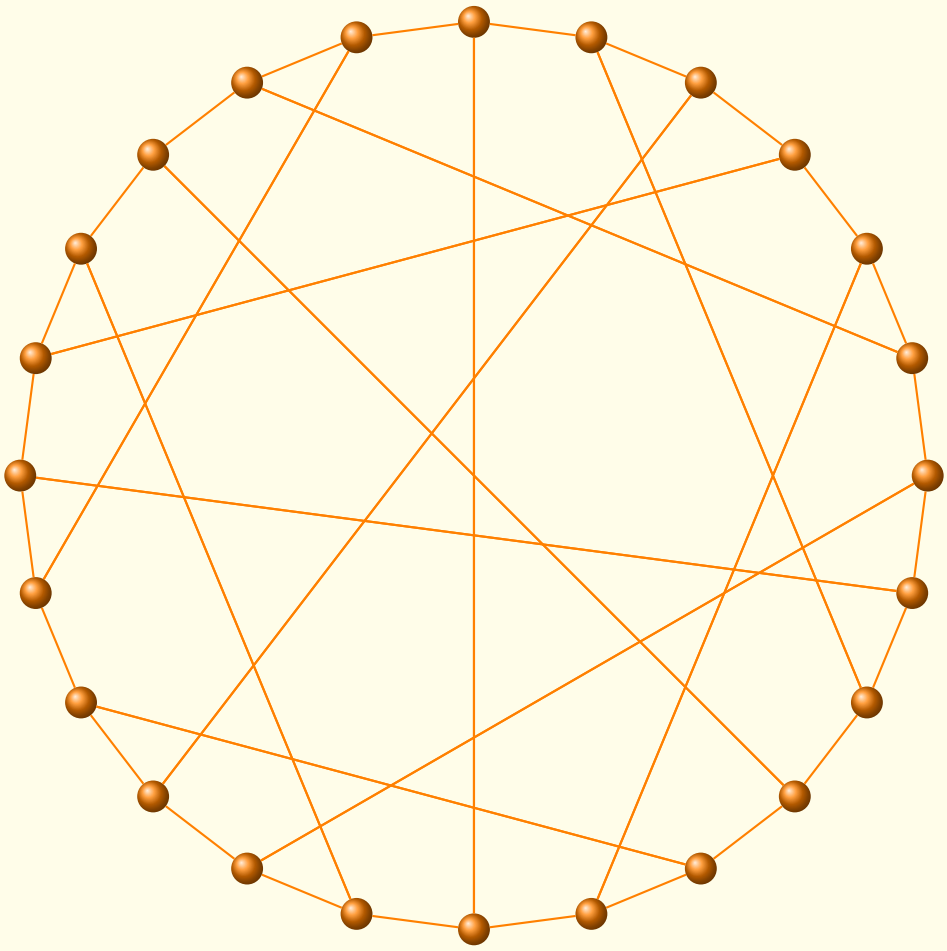
Others embeddings

22.2 McGee graph with \grLCF



```
\begin{tikzpicture}[rotate=90]
   \GraphInit[vstyle=Art]
    \grLCF[Math,RA=6]{-12,-6,6,-12,7,-7,-12,6,-6,-12,7,-7}{2}
\end{tikzpicture}
```

22.3 McGee graph with \grLCF



```
\begin{tikzpicture}[rotate=90]
   \GraphInit[vstyle=Art]
    \grLCF[Math,RA=6]{-12,6,-7,-12,7,-8,11,-6,6,-11,8,%
                      -7,-12,7,-6,-12,7,-11,-8,7,-7,8,11,-7}{1}
\end{tikzpicture}
```

SECTION 23

Möbius-Kantor Graph

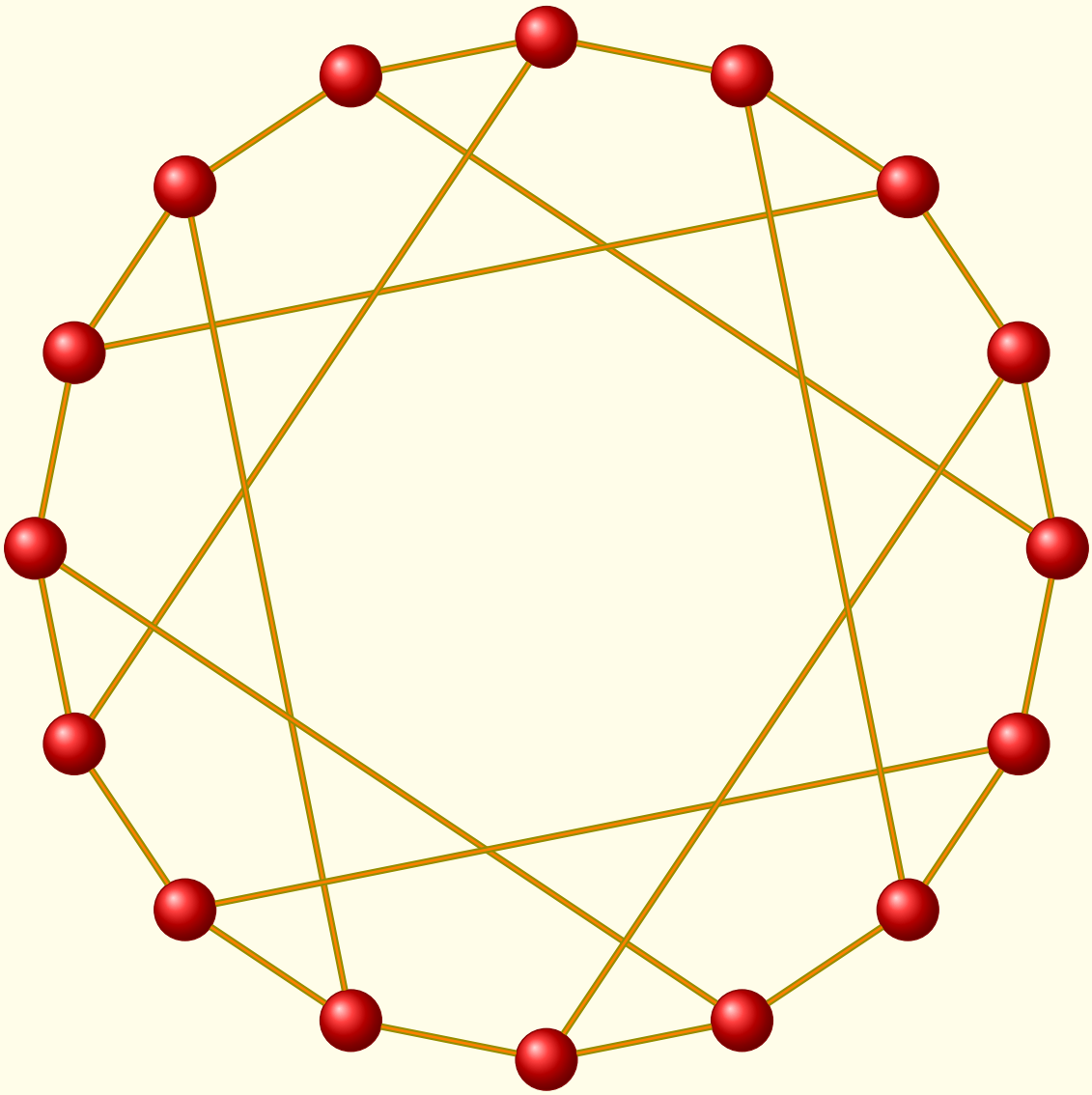
\grMobiusKantor[⟨*options*⟩]

From MathWord : http://mathworld.wolfram.com/Moebius-KantorGraph.html

The unique cubic symmetric graph on 16 nodes, illustrated above in several embeddings. It is 24 edges, girth 6, diameter 4, chromatic number 2, and is nonplanar but Hamiltonian. It can be represented in LCF notation and is identical to a generalized Petersen graph .

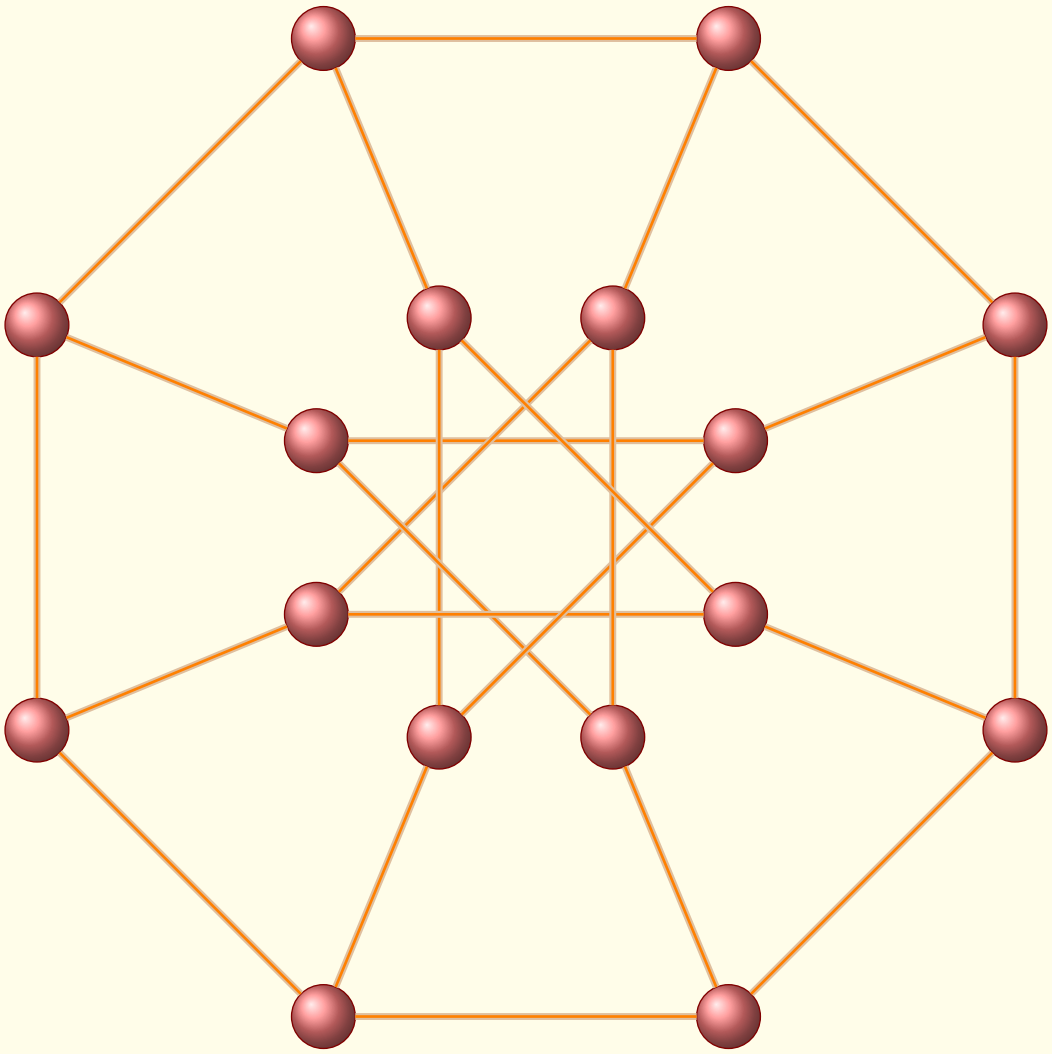
MathWorld by E.Weisstein

23.1 Möbius Graph : form I



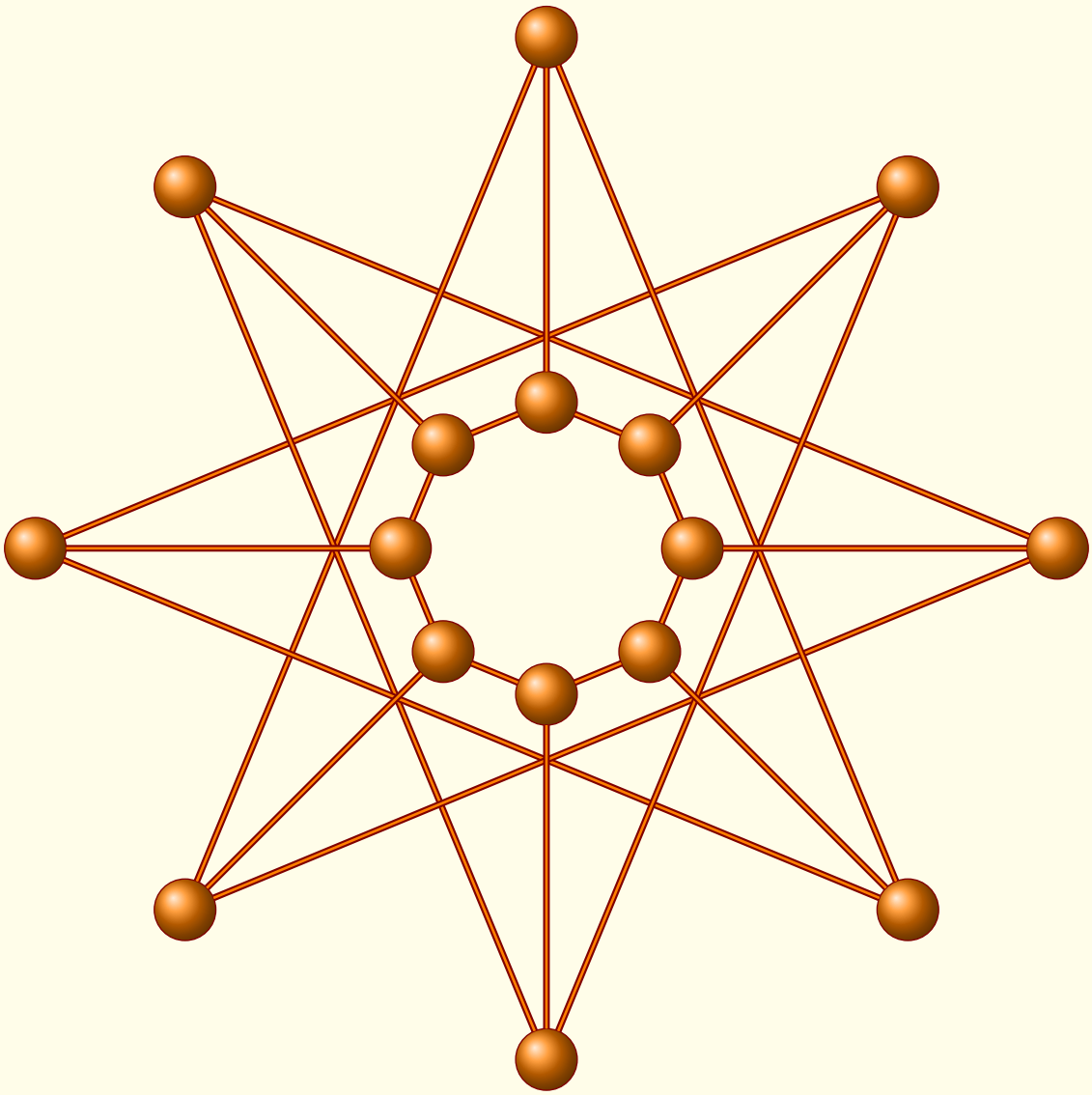
```
\begin{tikzpicture}
   \GraphInit[vstyle=Shade]
   \SetGraphArtColor{red}{olive}
   \SetVertexNoLabel
   \grMobiusKantor[RA=7]
\end{tikzpicture}
```

23.2 Möbius Graph : form II



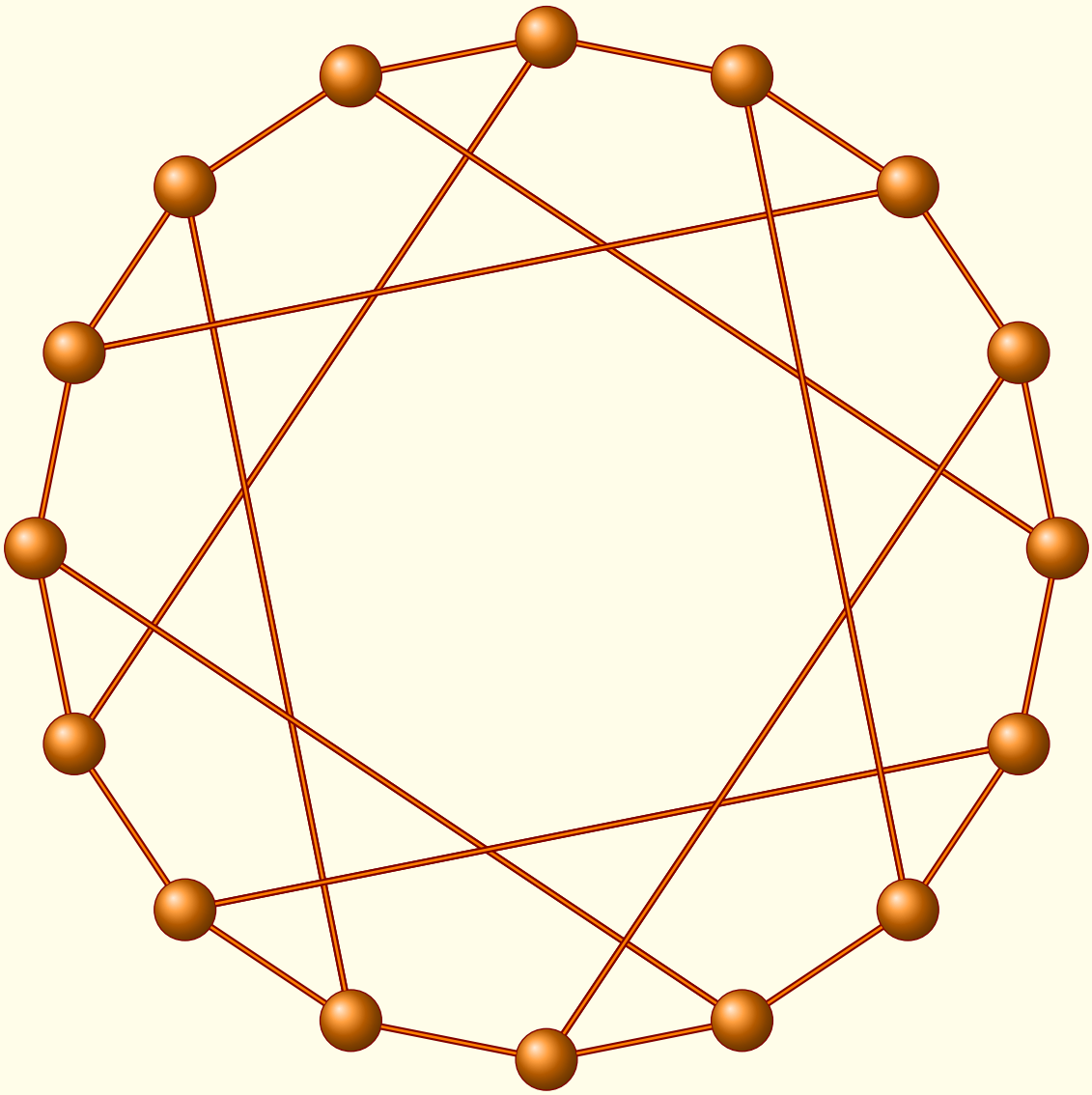
```
\begin{tikzpicture}[rotate=22.5]
   \GraphInit[vstyle=Shade]
  \SetGraphArtColor{red!50}{brown!50}
   \SetVertexNoLabel
   \grMobiusKantor[form=2,RA=7,RB=3]
\end{tikzpicture}
```

23.3 Möbius Graph : form III



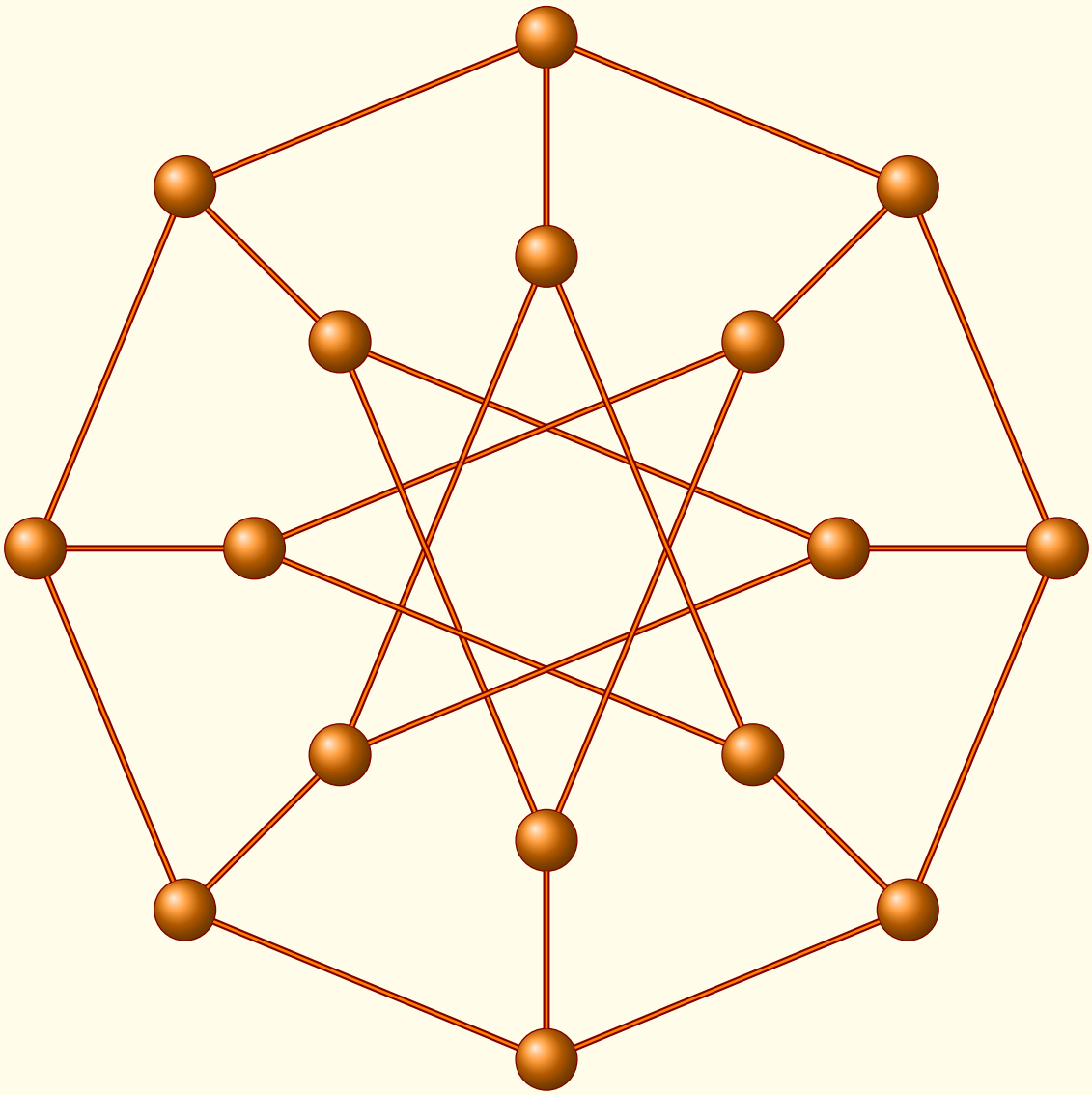
```
\begin{tikzpicture}
   \GraphInit[vstyle=Shade]
   \SetVertexNoLabel
   \grMobiusKantor[form=3,RA=7,RB=2]
\end{tikzpicture}
```

23.4 Möbius Graph with LCF notation



```
\begin{tikzpicture}[rotate=90]
   \GraphInit[vstyle=Shade]
   \SetVertexNoLabel
 \grLCF[RA=7]{5,-5}{8}
\end{tikzpicture}
```

23.5 Möbius Graph with \grGeneralizedPetersen

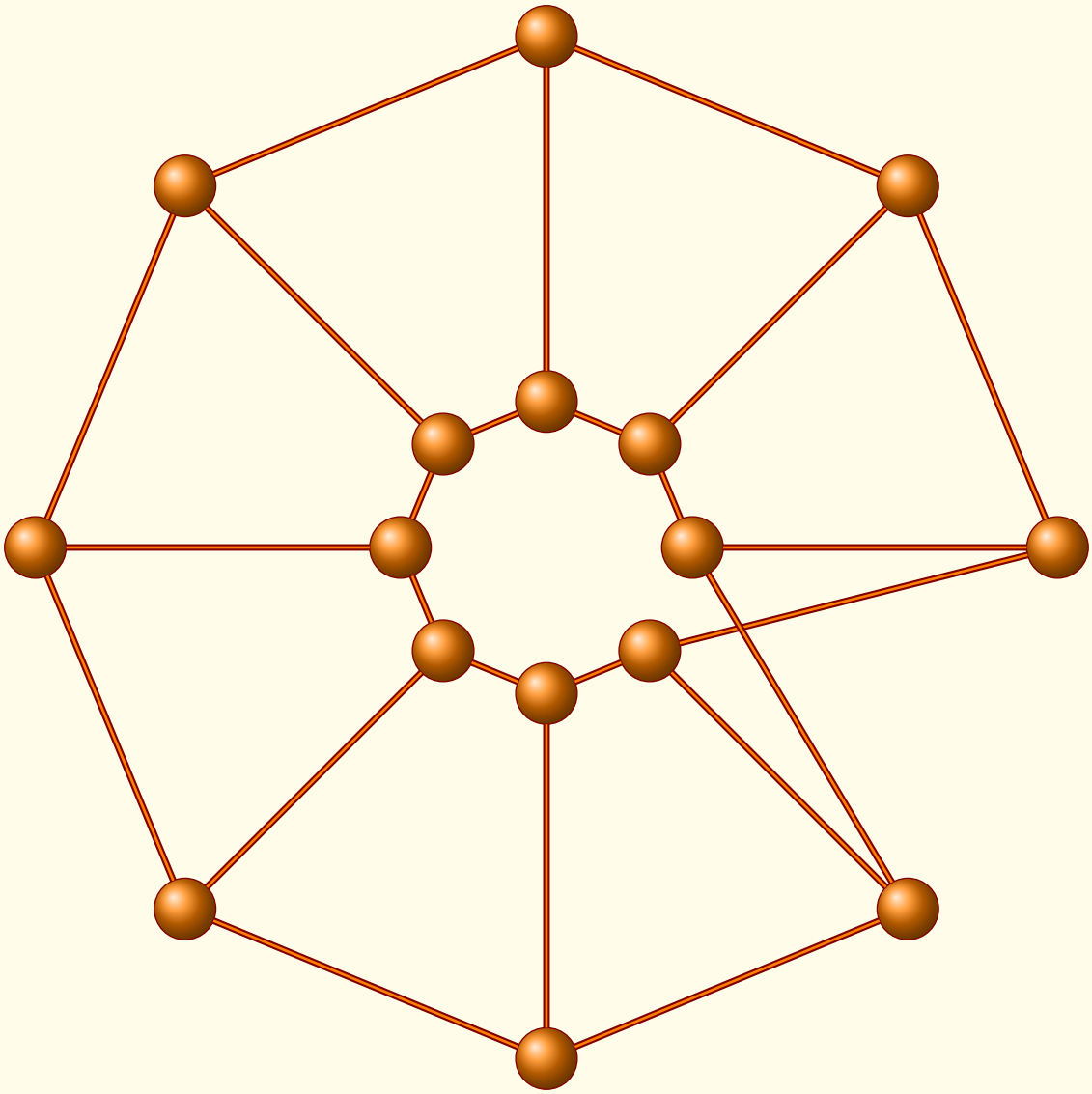


```
\begin{tikzpicture}
   \GraphInit[vstyle=Shade]
   \SetVertexNoLabel
  \grGeneralizedPetersen[RA=7,RB=4]{8}{3}
\end{tikzpicture}
```

A Möbius ladder of order $2n$ is a simple graph obtained by introducing a twist in a prism graph of order $2n$ that is isomorphic to the circulant graph with order $2n$ and $L = \{1, n\}$

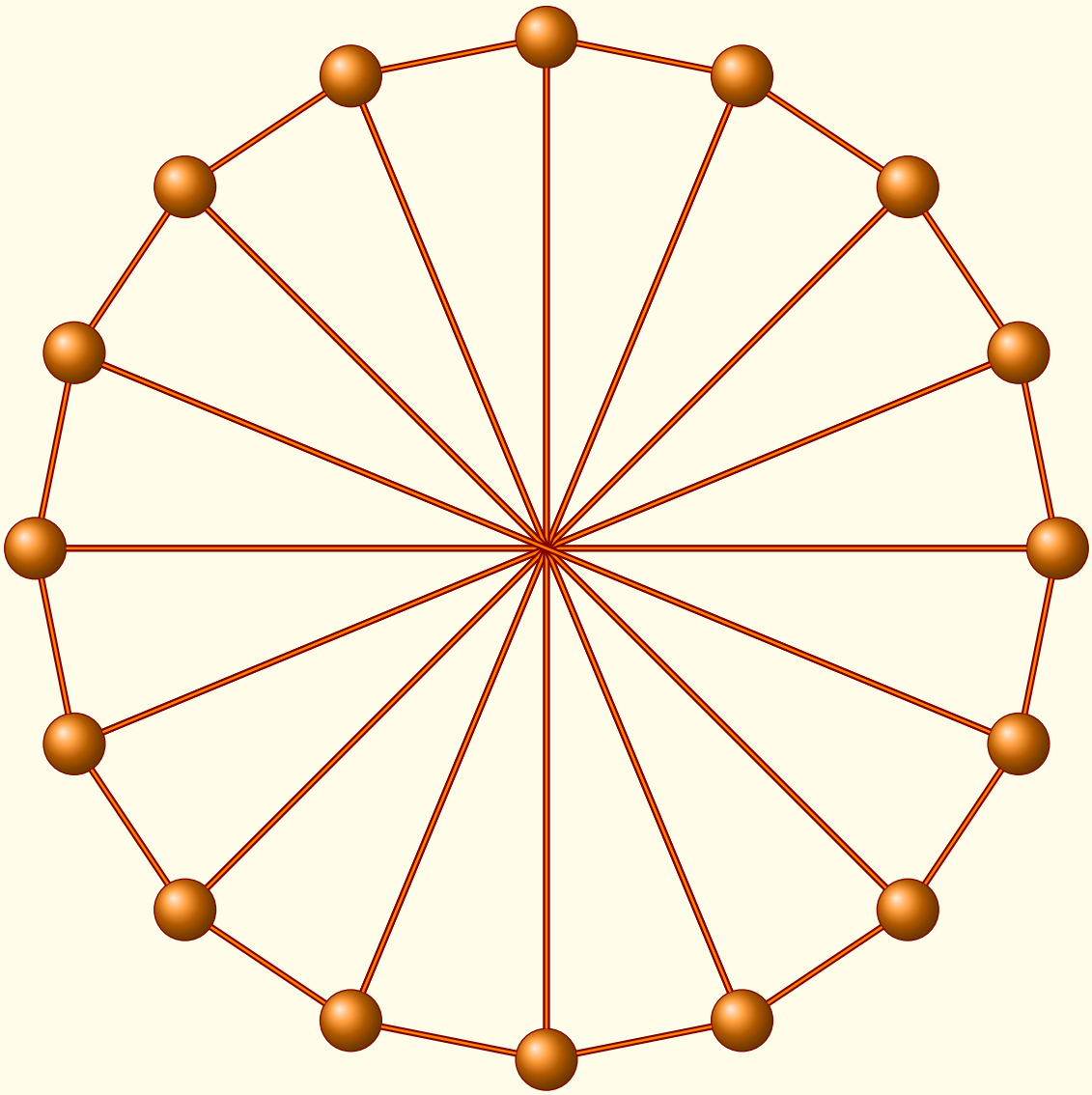
http://mathworld.wolfram.com/MoebiusLadder.html

23.6 Möbius Ladder Graph



```
\begin{tikzpicture}
   \GraphInit[vstyle=Shade]
   \grMobiusLadder[RA=7,RB=2]{8}%
\end{tikzpicture}
```

23.7 Circulant Graph isomorphic to the last graph



```
\begin{tikzpicture}
   \GraphInit[vstyle=Shade]
   \grCirculant[RA=7]{16}{1,8}%
\end{tikzpicture}
```

SECTION 24

Pappus

\grPappus[⟨*options*⟩]

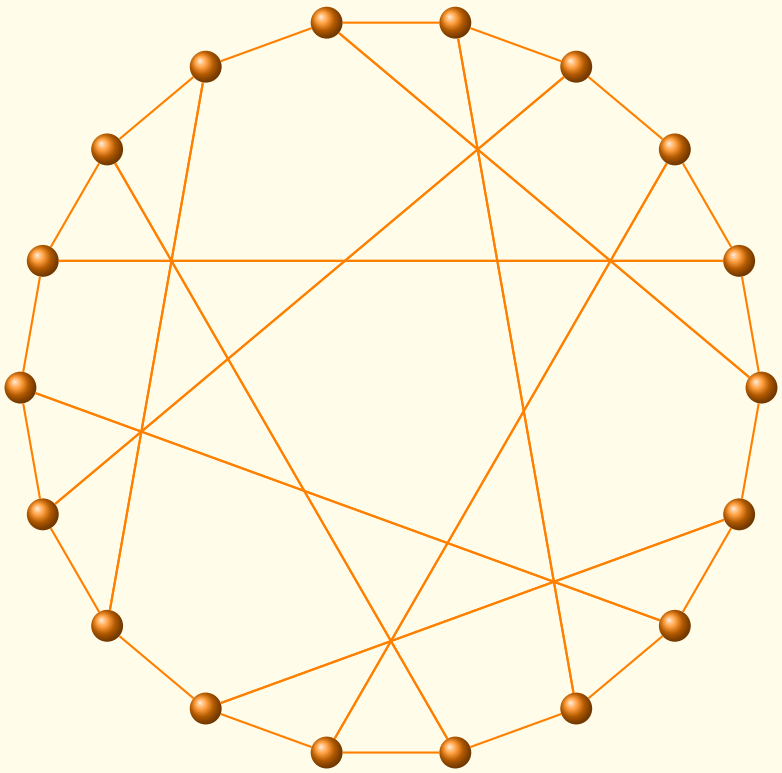
From MathWord : http://mathworld.wolfram.com/PappusGraph.html

A cubic symmetric distance-regular graph on 18 vertices, illustrated below in three embeddings. It can be represented in LCF notation $[5,7,-7,7,-7,-5]^3$ *(Frucht 1976).* MathWorld by E.Weisstein

From Wikipedia : http://en.wikipedia.org/wiki/Pappus_graph *In the mathematical field of graph theory, the Pappus graph is a 3-regular graph with 18 vertices and 27 edges, formed as the Levi graph of the Pappus configuration. It is a distance-regular graph, one of only 14 such cubic graphs according to Cubic symmetric graphs.*

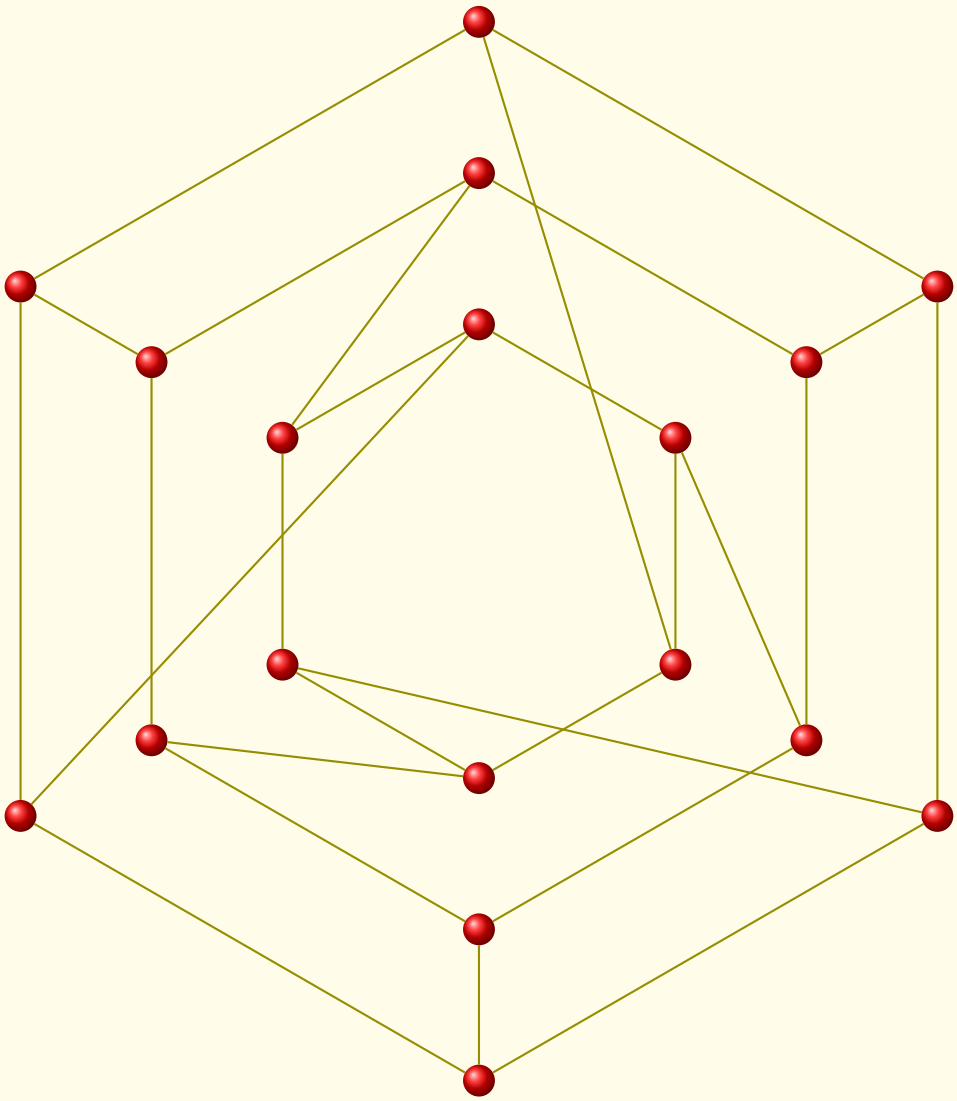
This macro can be used with three different forms.

24.1 Pappus Graph : form 1



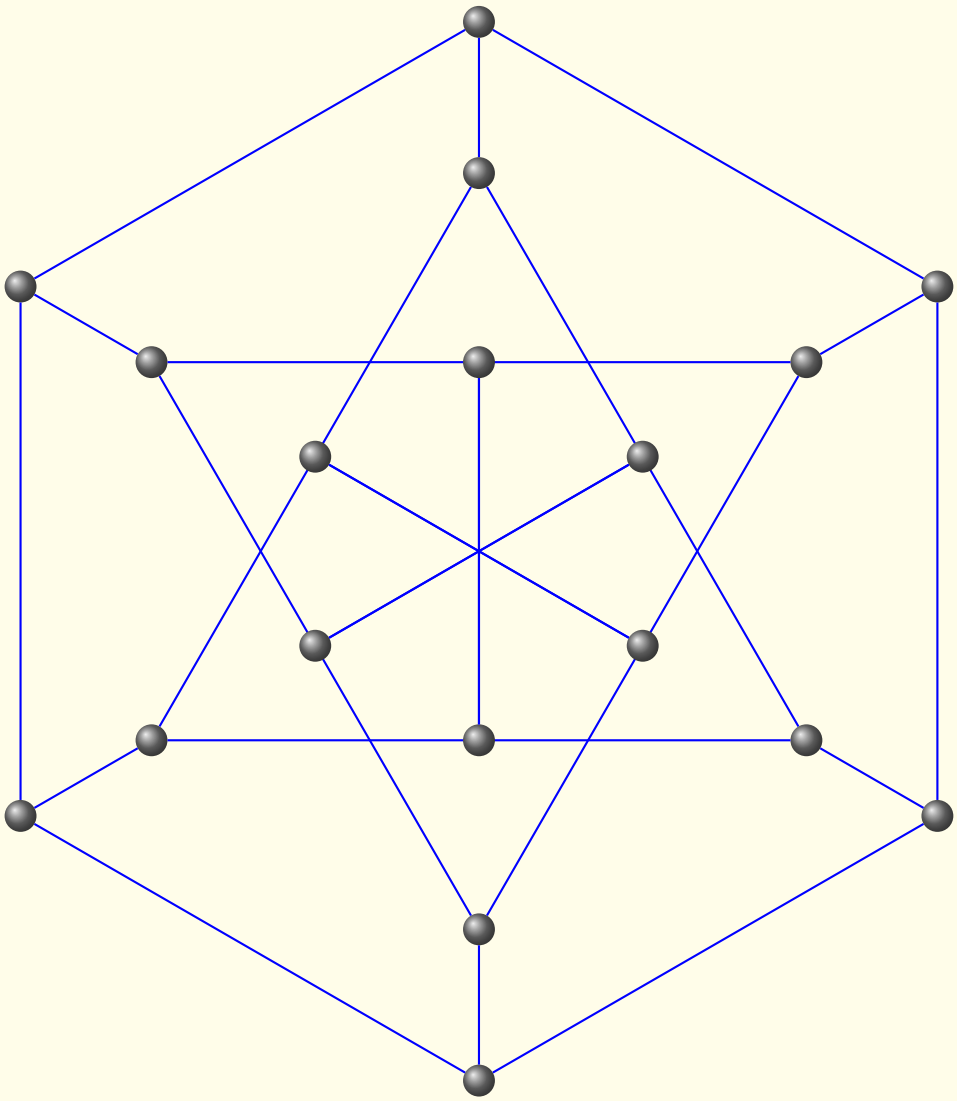
```
\begin{tikzpicture}[scale=.7]
   \GraphInit[vstyle=Art]
   \grPappus[RA=7]
\end{tikzpicture}
```

24.2 Pappus Graph : form 2



```
\begin{tikzpicture}
   \GraphInit[vstyle=Art]
   \SetGraphArtColor{red}{olive}
   \grPappus[form=2,RA=7,RB=5,RC=3]
\end{tikzpicture}
```

24.3 Pappus Graph : form 3



```
\begin{tikzpicture}
   \GraphInit[vstyle=Art]
   \SetGraphArtColor{gray}{blue}
   \grPappus[form=3,RA=7,RB=5,RC=2.5]
\end{tikzpicture}
```

SECTION 25

Petersen

\grPetersen[⟨*options*⟩]

From MathWord : http://mathworld.wolfram.com/PetersenGraph.html

The Petersen graph is the graph , illustrated below in several embeddings, possessing 10 nodes, all of whose nodes have degree three. The Petersen graph is implemented in ***tkz-berge*** *as* ***\grPetersen****. The Petersen graph has girth 5, diameter 2, edge chromatic number 4, chromatic number 3.*

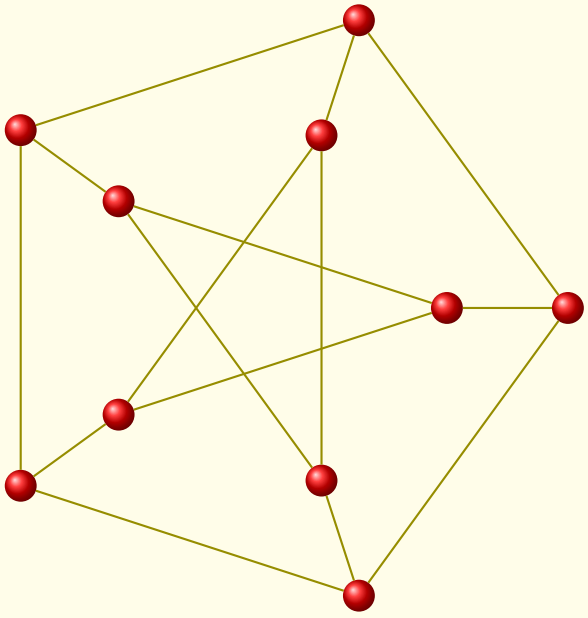
MathWorld by E.Weisstein

From Wikipedia : http://en.wikipedia.org/wiki/Petersen_graph

In graph theory, the Petersen graph is an undirected graph with 10 vertices and 15 edges. It is a small graph that serves as a useful example and counterexample for many problems in graph theory. The Petersen graph is named for Julius Petersen, who in 1898 constructed it to be the smallest bridgeless cubic graph with no three-edge-coloring. Although the graph is generally credited to Petersen, it had in fact first appeared 12 years earlier, in 1886.

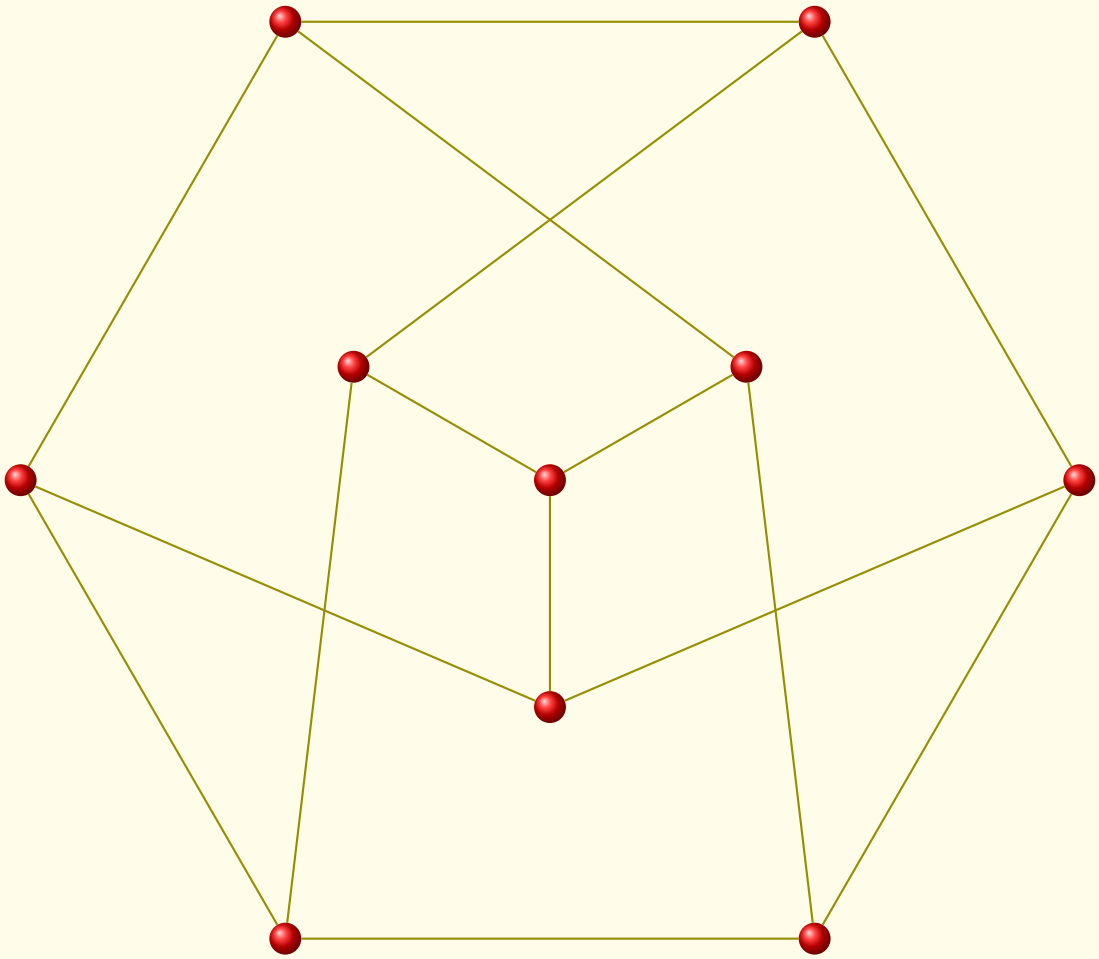
This macro can be used with three different forms.

25.1 Petersen graph : form 1



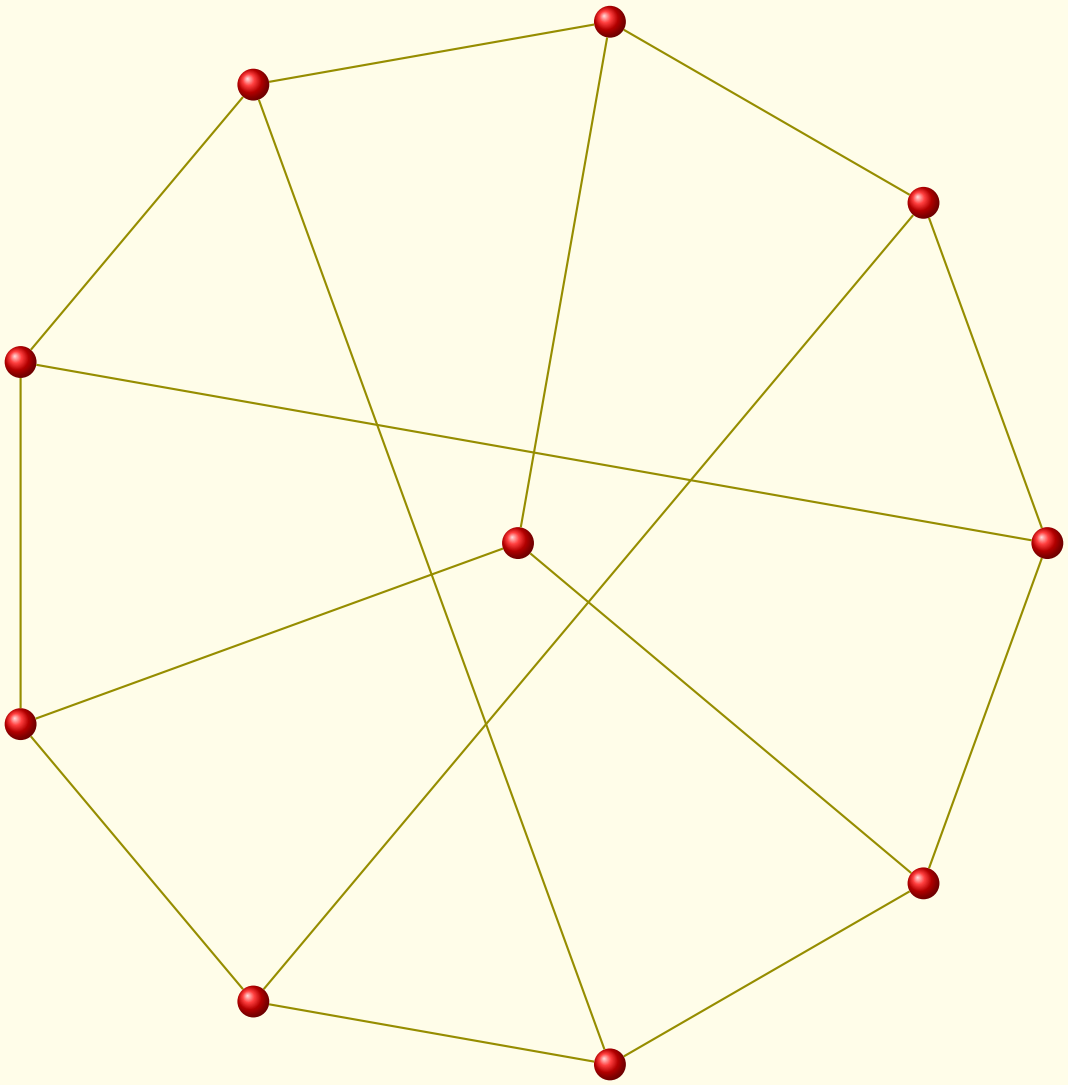
```
\begin{tikzpicture}[scale=.8]
    \GraphInit[vstyle=Art]
    \SetGraphArtColor{red}{olive}
    \grPetersen[form=1,RA=5,RB=3]%
\end{tikzpicture}
```

25.2 Petersen graph : form 2



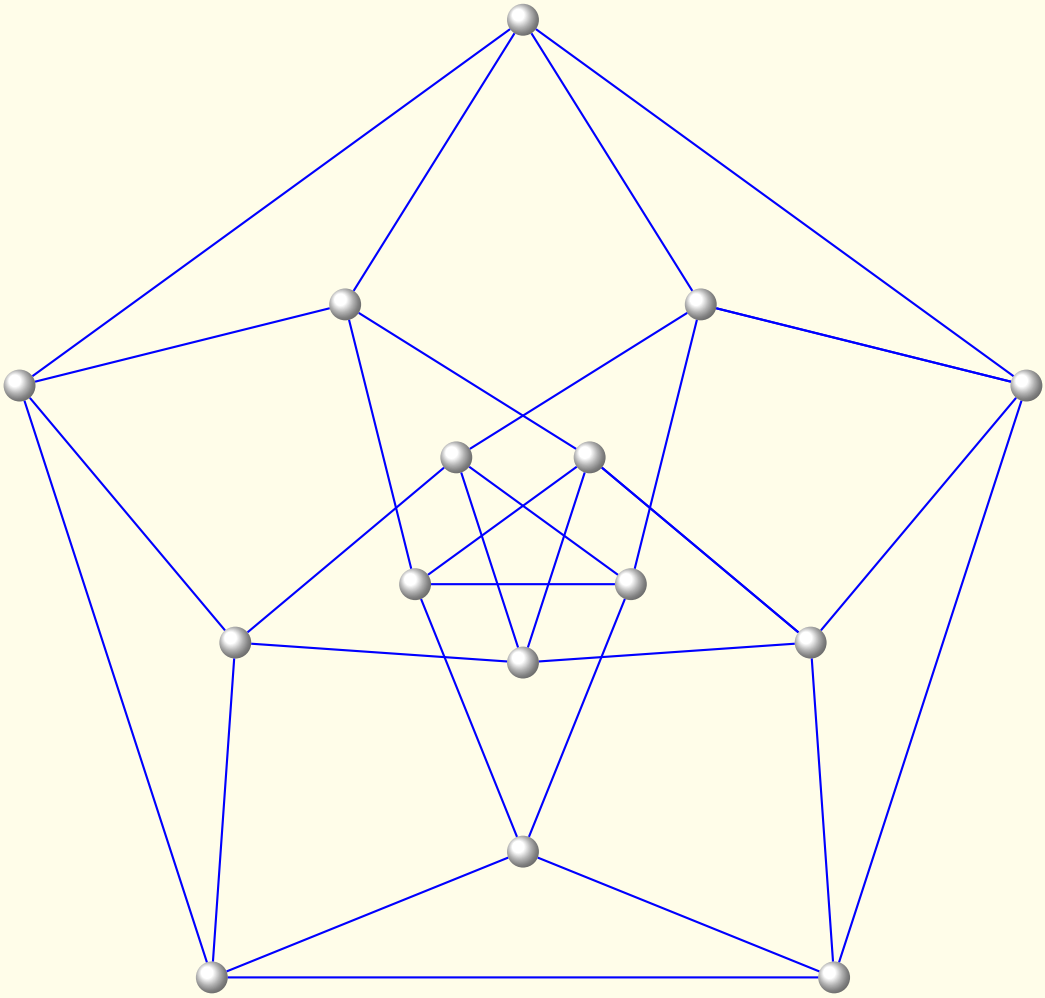
```
\begin{tikzpicture}%
   \GraphInit[vstyle=Art]
   \SetGraphArtColor{red}{olive}
   \grPetersen[form=2,RA=7,RB=3]%
\end{tikzpicture}
```

25.3 Petersen graph : form 3



```
\begin{tikzpicture}%
    \GraphInit[vstyle=Art]
    \SetGraphArtColor{red}{olive}
    \grPetersen[form=3,RA=7]%
\end{tikzpicture}
```

25.4 The line graph of the Petersen graph



```
\begin{tikzpicture}
\GraphInit[vstyle=Art]\SetGraphArtColor{white}{blue}
  \begin{scope}[rotate=-90] \grCirculant[RA=1.5,prefix=a]{5}{2}\end{scope}
  \begin{scope}[rotate=-18] \grEmptyCycle[RA=4,prefix=b]{5}{2} \end{scope}
  \begin{scope}[rotate=18]  \grCycle[RA=7,prefix=c]{5}         \end{scope}
  \EdgeIdentity{a}{b}{5}
  \EdgeIdentity{b}{c}{5}
  \EdgeDoubleMod{b}{5}{0}{1}{a}{5}{2}{1}{5}
  \EdgeDoubleMod{c}{5}{0}{1}{b}{5}{1}{1}{5}
\end{tikzpicture}
```

\grGeneralizedPetersen[⟨*RA=⟨Number⟩,RB=⟨Number⟩*⟩]{⟨*integer*⟩}{⟨*integer*⟩}

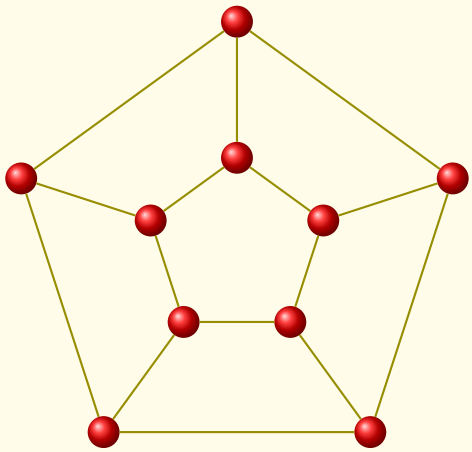
From MathWord : http://mathworld.wolfram.com/GeneralizedPetersenGraph.html

The generalized Petersen graph , also denoted GP(n,k) *, for* $n \geq 3$ *and* $1 \leq k \leq \lfloor (n-1)/2 \rfloor$ *is a graph consisting of an inner star polygon (circulant graph) and an outer regular polygon (cycle graph) with corresponding vertices in the inner and outer polygons connected with edges. has nodes and edges. The Petersen graph is implemented in* ***tkz-berge*** *as* **\grGeneralizedPetersen**. MathWorld by E.Weisstein

From Wikipedia : http://en.wikipedia.org/wiki/Petersen_graph *In 1950 H. S. M. Coxeter introduced a family of graphs generalizing the Petersen graph. These graphs are now called generalized Petersen graphs, a name given to them in 1969 by Mark Watkins. In Watkins' notation,* G(n,k) *is a graph with vertex set*
$u_0, u_1, \ldots, u_{n-1}, v_0, v_1, \ldots, v_{n-1}$
and edge set
$u_i u_{i+1}, u_i v_i, v_i u_{i+k} : i = 0, \ldots, n-1$
where subscripts are to be read modulo n *and* $k < n/2$*. Coxeter's notation for the same graph would be* $\{n\} + \{n/k\}$*. The Petersen Graph itself is* G$(5,2)$ *or* $\{5\} + \{5/2\}$.

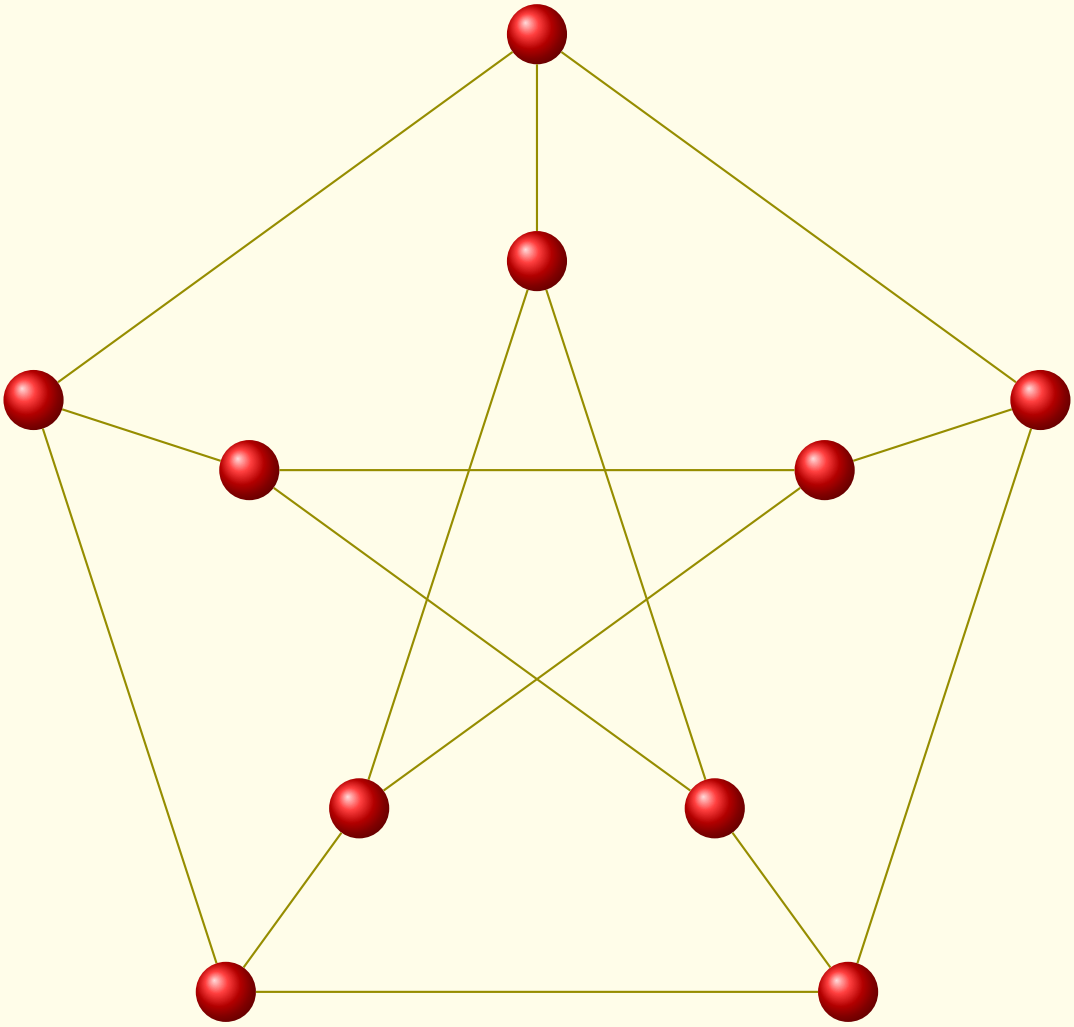
This macro can be used with three different forms.

25.5 Generalized Petersen graph GP(5,1)



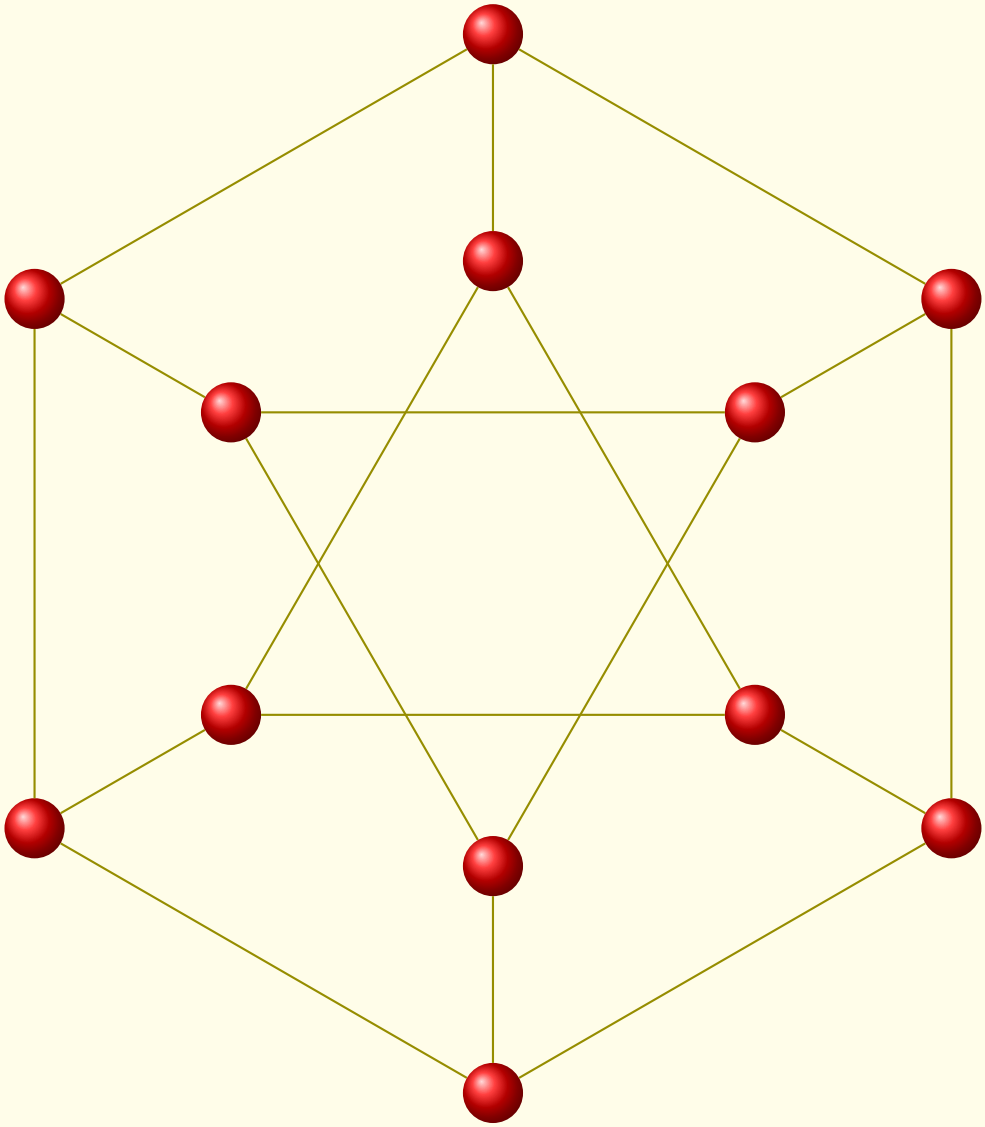
```
\begin{tikzpicture}[rotate=90,scale=.6]
  \GraphInit[vstyle=Art]\SetGraphArtColor{red}{olive}
  \renewcommand*{\VertexInnerSep}{4pt}
  \grGeneralizedPetersen[RA=5,RB=2]{5}{1}
\end{tikzpicture}
```

25.6 The Petersen graph GP(5,2)



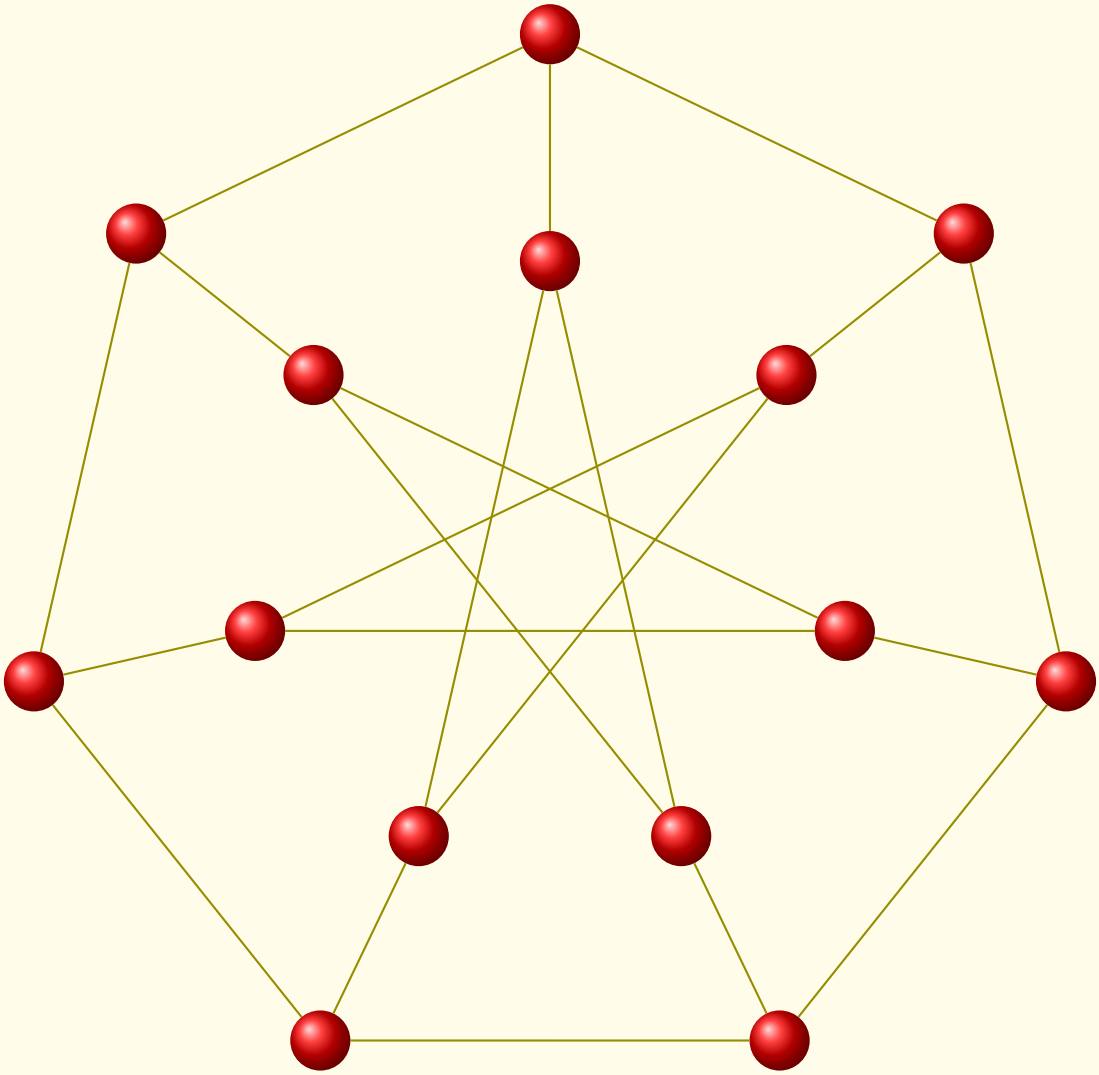
```
\begin{tikzpicture}[rotate=90]
    \GraphInit[vstyle=Art]\SetGraphArtColor{red}{olive}
    \renewcommand*{\VertexInnerSep}{8pt}
    \grGeneralizedPetersen[RA=7,RB=4]{5}{2}
\end{tikzpicture}
```

25.7 Generalized Petersen graph GP(6,2)



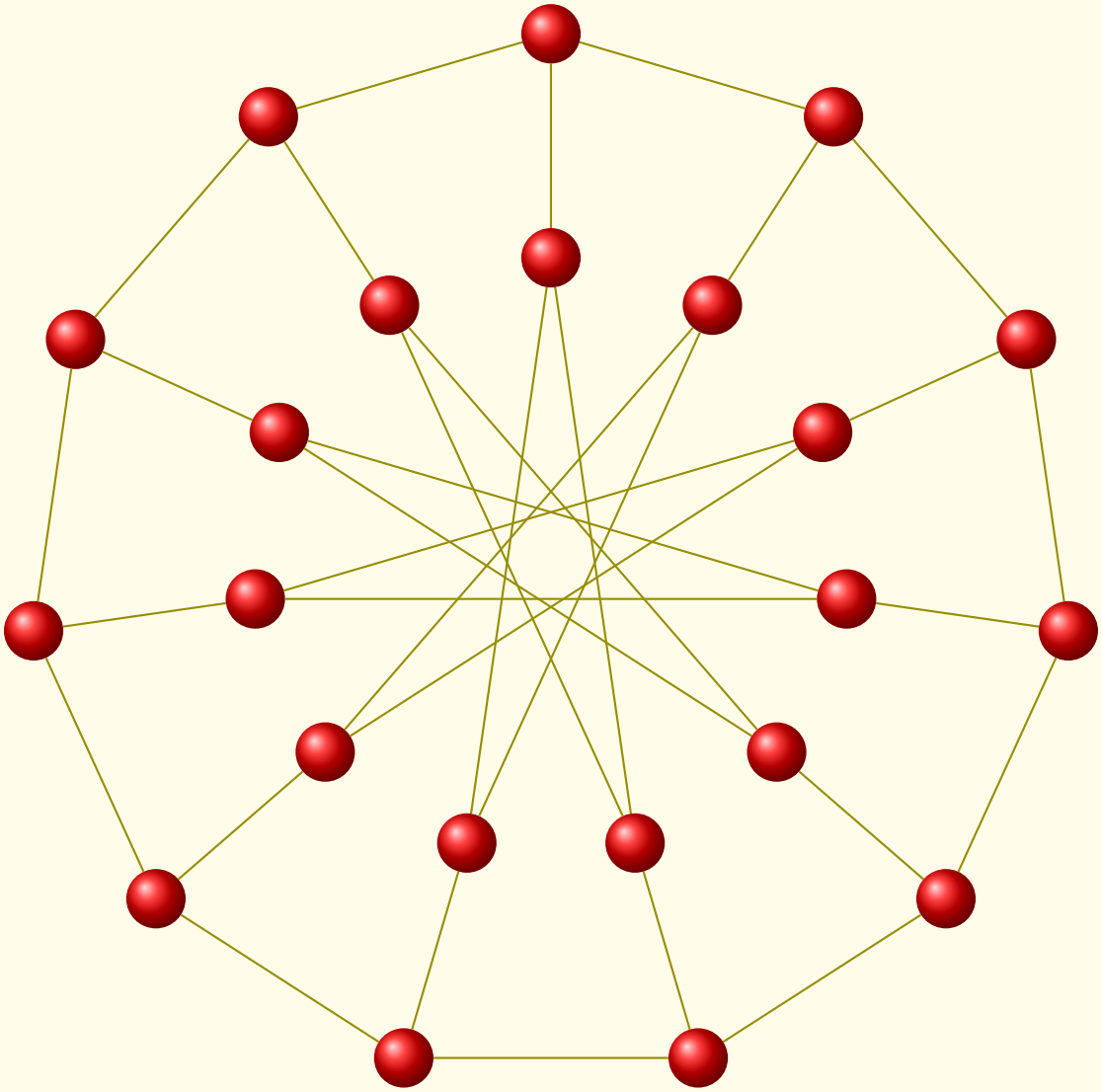
```
\begin{tikzpicture}[rotate=90]
  \GraphInit[vstyle=Art]\SetGraphArtColor{red}{olive}
  \renewcommand*{\VertexInnerSep}{8pt}
  \grGeneralizedPetersen[RA=7,RB=4]{6}{2}
\end{tikzpicture}
```

25.8 Generalized Petersen graph GP(7,3)



```
\begin{tikzpicture}[rotate=90]
  \GraphInit[vstyle=Art]\SetGraphArtColor{red}{olive}
  \renewcommand*{\VertexInnerSep}{8pt}
  \grGeneralizedPetersen[RA=7,RB=4]{7}{3}
\end{tikzpicture}
```

25.9 Generalized Petersen graph GP(11,5)



```
\begin{tikzpicture}[rotate=90]
  \renewcommand*{\VertexInnerSep}{8pt}
  \GraphInit[vstyle=Art]\SetGraphArtColor{red}{olive}
  \grGeneralizedPetersen[RA=7,RB=4]{11}{5}
\end{tikzpicture}
```

SECTION 26

The five Platonics Graphs

The Platonic Graphs are the graphs formed by the edges and vertices of the five regular Platonic solids. The five Platonics Graphs are illustrated below.

1. tetrahedral
2. octahedral
3. cube
4. icosahedral
5. dodecahedral

\grTetrahedral[⟨*RA=Number*⟩]

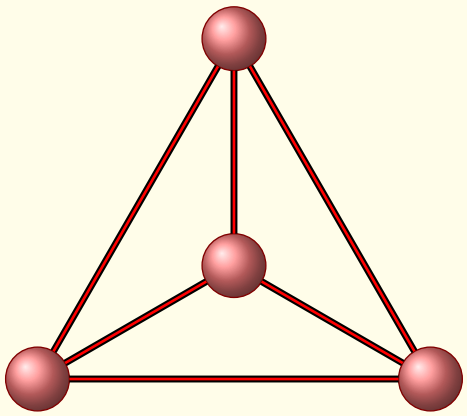
From MathWord : http://mathworld.wolfram.com/TetrahedralGraph.html

Tetrahedral Graph *is the unique polyhedral graph on four nodes which is also the complete graph and therefore also the wheel graph . It is implemented as* ***\grTetrahedral*** MathWorld by E.Weisstein It has :

1. 4 nodes,
2. 6 edges,
3. graph diameter 1.

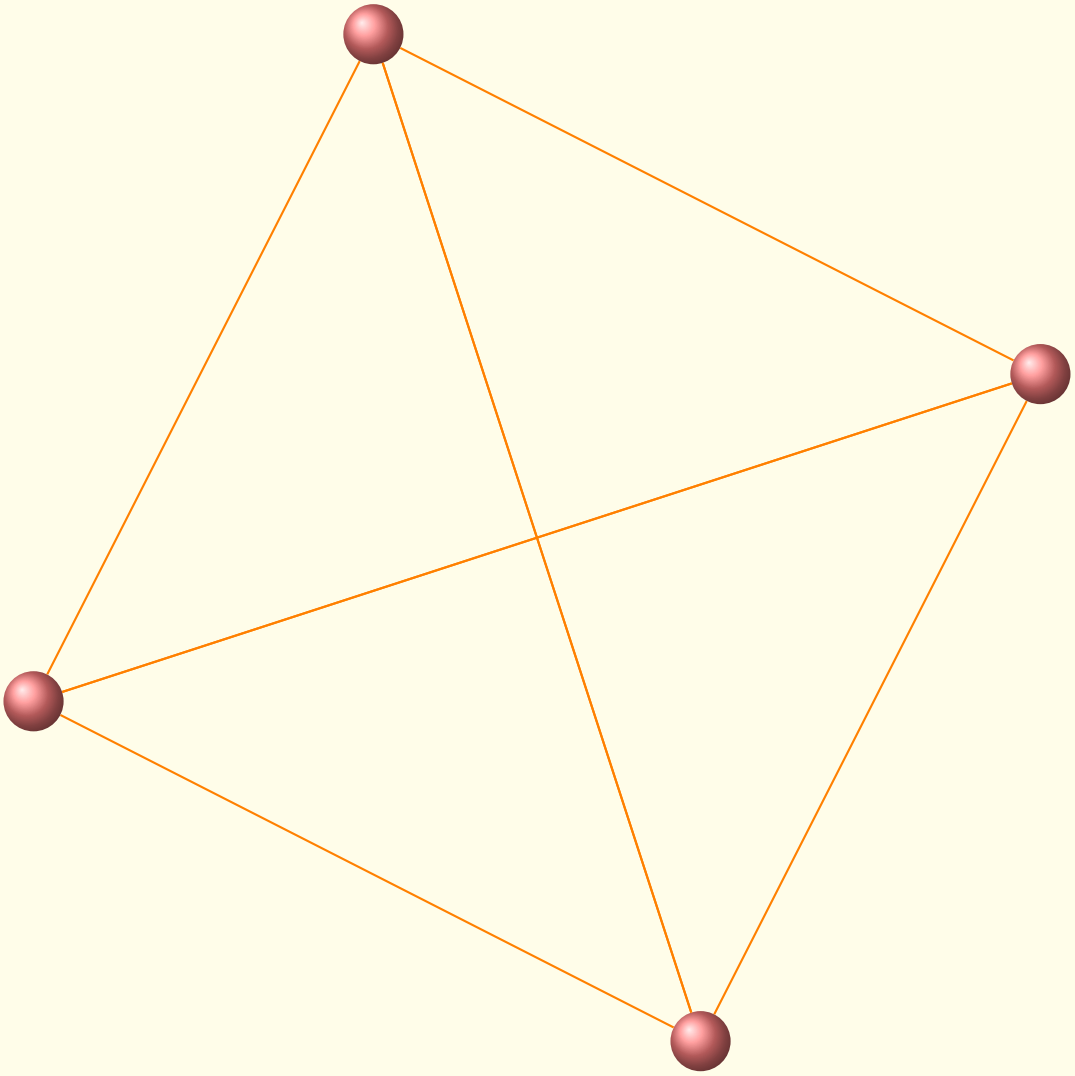
The Tetrahedral Graph is 3-Regular

26.1 Tetrahedral



```
\begin{tikzpicture}[scale=.6]
      \GraphInit[vstyle=Shade]
      \renewcommand*{\VertexInnerSep}{4pt}
      \SetVertexNoLabel\SetGraphShadeColor{red!50}{black}{red}
      \grTetrahedral[RA=5]
\end{tikzpicture}
```

26.2 Tetrahedral LCF embedding



```
\begin{tikzpicture}[rotate=18]
     \renewcommand*{\VertexInnerSep}{8pt}
     \GraphInit[vstyle=Art]
     \SetGraphArtColor{red!50}{orange}
     \grLCF[RA=7]{2,-2}{2}
\end{tikzpicture}
```

`\grOctahedral[⟨RA=⟨Number⟩,RB=⟨Number⟩⟩]`

From MathWord : http://mathworld.wolfram.com/OctahedralGraph.html

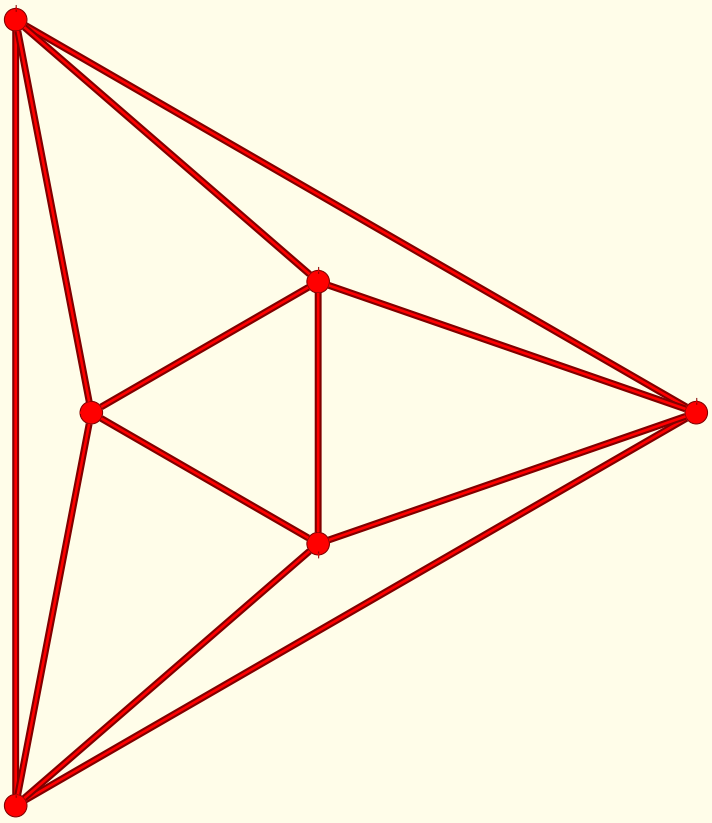
Octahedral Graph *is isomorphic to the circulant graph* $\mathrm{CI}_{[1,2]}(6)$ *. Two embeddings of this graph are illustrated below. It is implemented as* ***\grOctahedral*** *or as* ***\grSQCycle{6}***. MathWorld by E.Weisstein

It has :

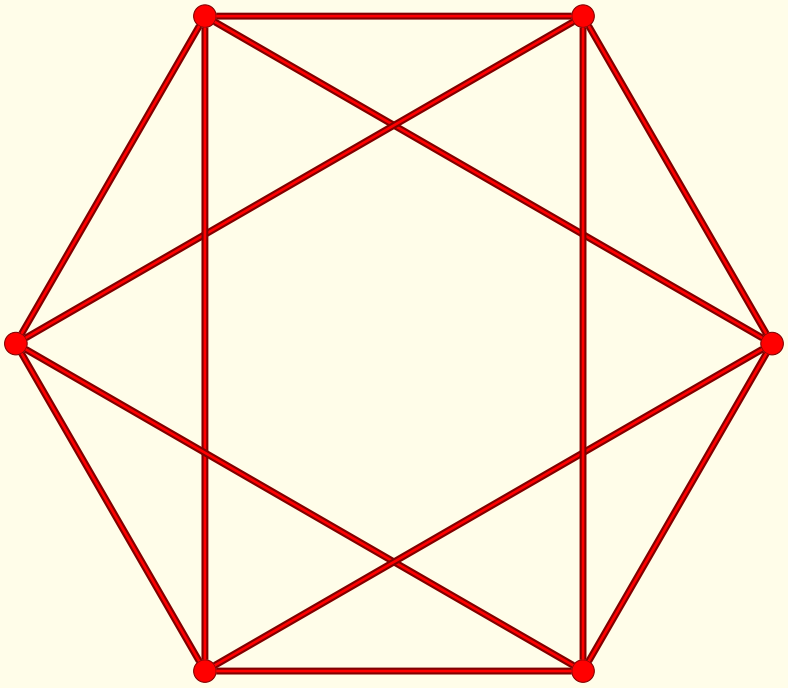
1. 6 nodes,
2. 12 edges,
3. graph diameter 2.

The Octahedral Graph is 4-Regular.

26.3 Octahedral



```
\begin{tikzpicture}
  \grOctahedral[RA=6,RB=2]
\end{tikzpicture}
```



```
\begin{tikzpicture}
       \grSQCycle[RA=5]{6}
\end{tikzpicture}
```

`\grCubicalGraph[⟨RA=⟨Number⟩,RB=⟨Number⟩⟩]`

From MathWord : http://mathworld.wolfram.com/CubicalGraph.html

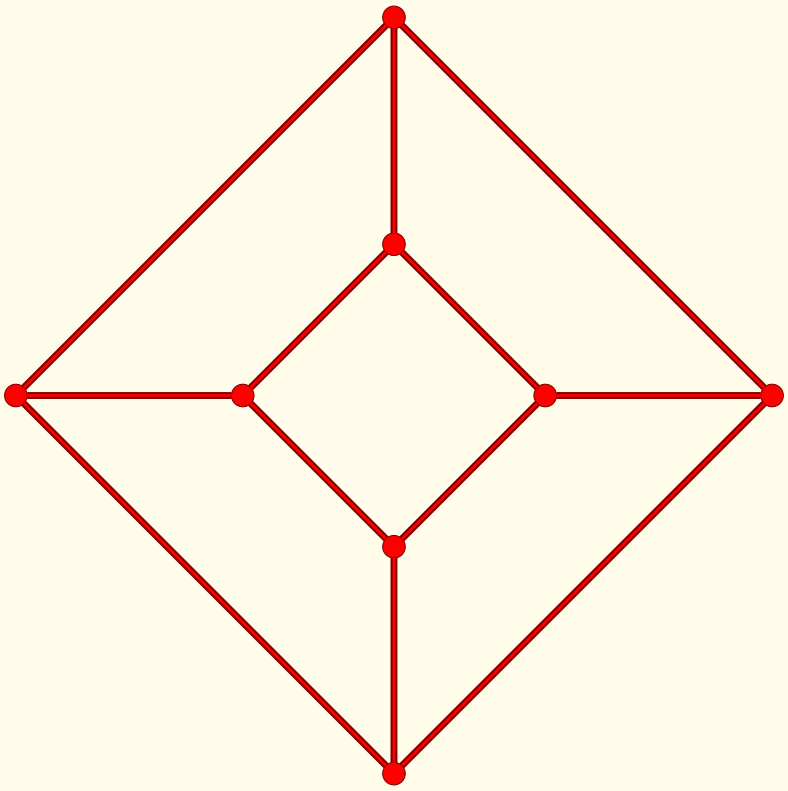
Cubical Graph *is isomorphic to a generalized Petersen graph* $PG_{[4,1]}$*, to a bipartite Kneser graph , to a crown graph and it is equivalent to the Cycle Ladder* CL(4)*. Two embeddings of this graph are illustrated below. It is implemented as* ***\grCubicalGraph*** *or* ***\grPrism{4}***. MathWorld by E.Weisstein

It has :

1. 8 nodes,
2. 12 edges,
3. graph diameter 3.

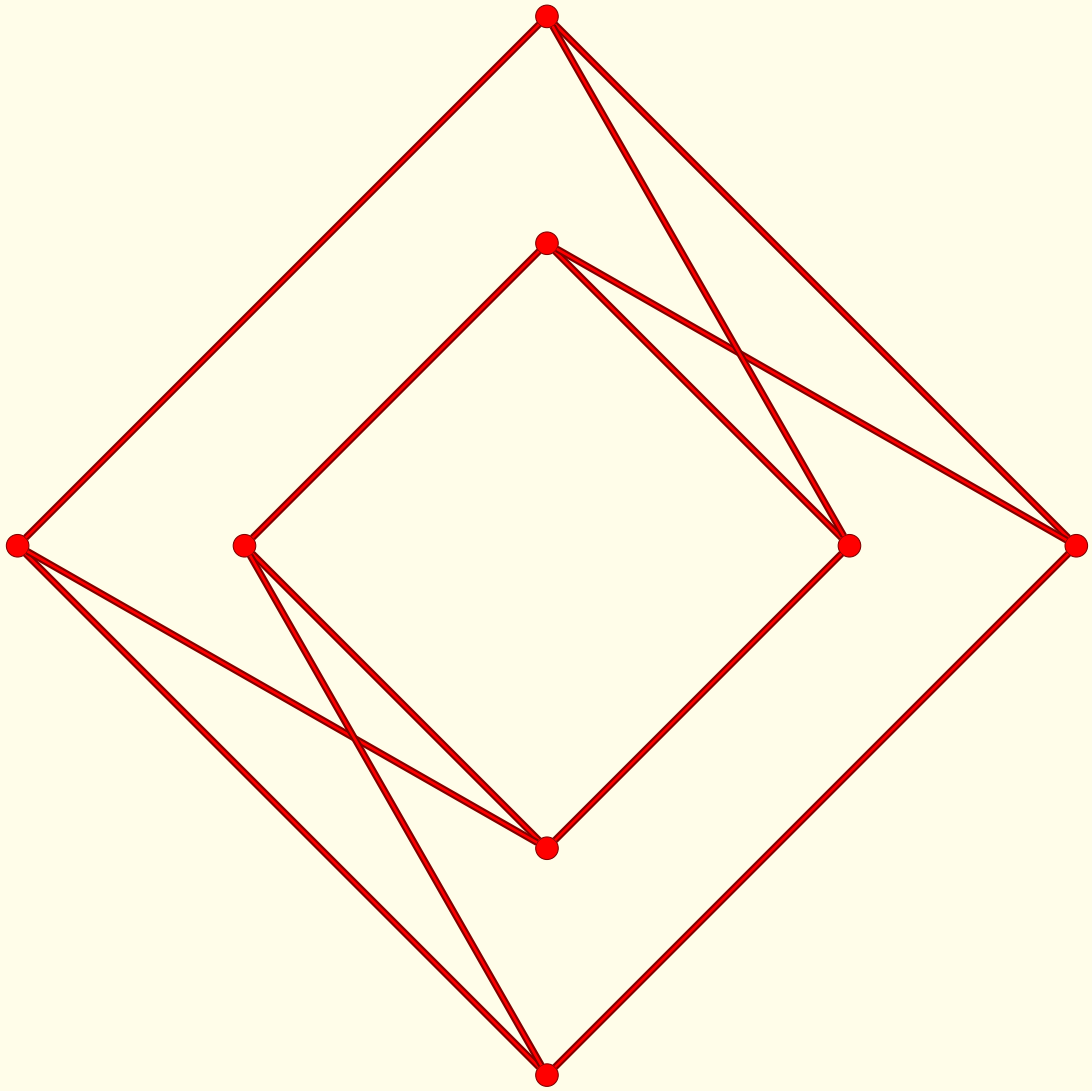
The Cubical Graph is 3-Regular.

26.4 Cubical Graph : form 1



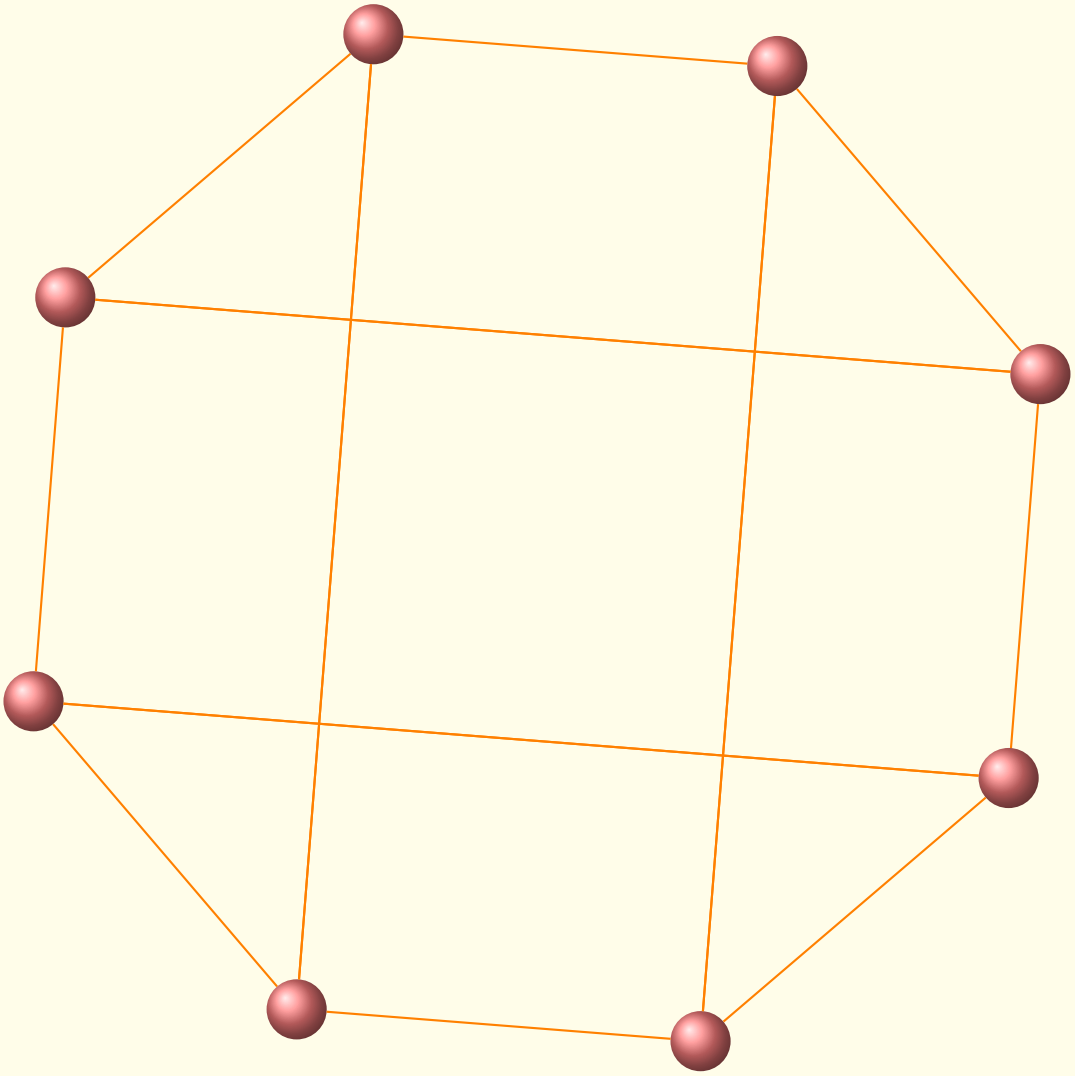
```
\begin{tikzpicture}
  \grCubicalGraph[RA=5,RB=2]
\end{tikzpicture}
```

26.5 Cubical Graph : form 2



```
\begin{tikzpicture}
  \grCubicalGraph[form=2,RA=7,RB=4]
\end{tikzpicture}
```

26.6 Cubical LCF embedding



```
\begin{tikzpicture}[rotate=18]
    \GraphInit[vstyle=Art]\renewcommand*{\VertexInnerSep}{8pt}
    \SetGraphArtColor{red!50}{orange}
    \grLCF[RA=7]{3,-3}{4}
\end{tikzpicture}
```

`\grIcosahedral[⟨RA=⟨Number⟩,RB=⟨Number⟩,RC=⟨Number⟩⟩]`

From MathWord : http://mathworld.wolfram.com/IcosahedralGraph.html

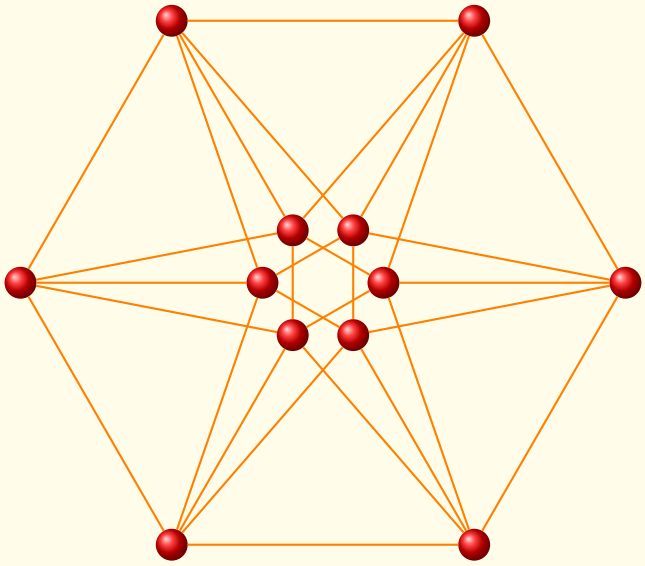
The ***Icosahedral Graph*** *is the Platonic graph whose nodes have the connectivity of the icosahedron, illustrated above in a number of embeddings. The icosahedral graph has 12 vertices and 30 edges. Since the icosahedral graph is regular and Hamiltonian, it has a generalized LCF notation.* MathWorld by E.Weisstein

It has :

1. 12 nodes,
2. 30 edges,
3. graph diameter 3.

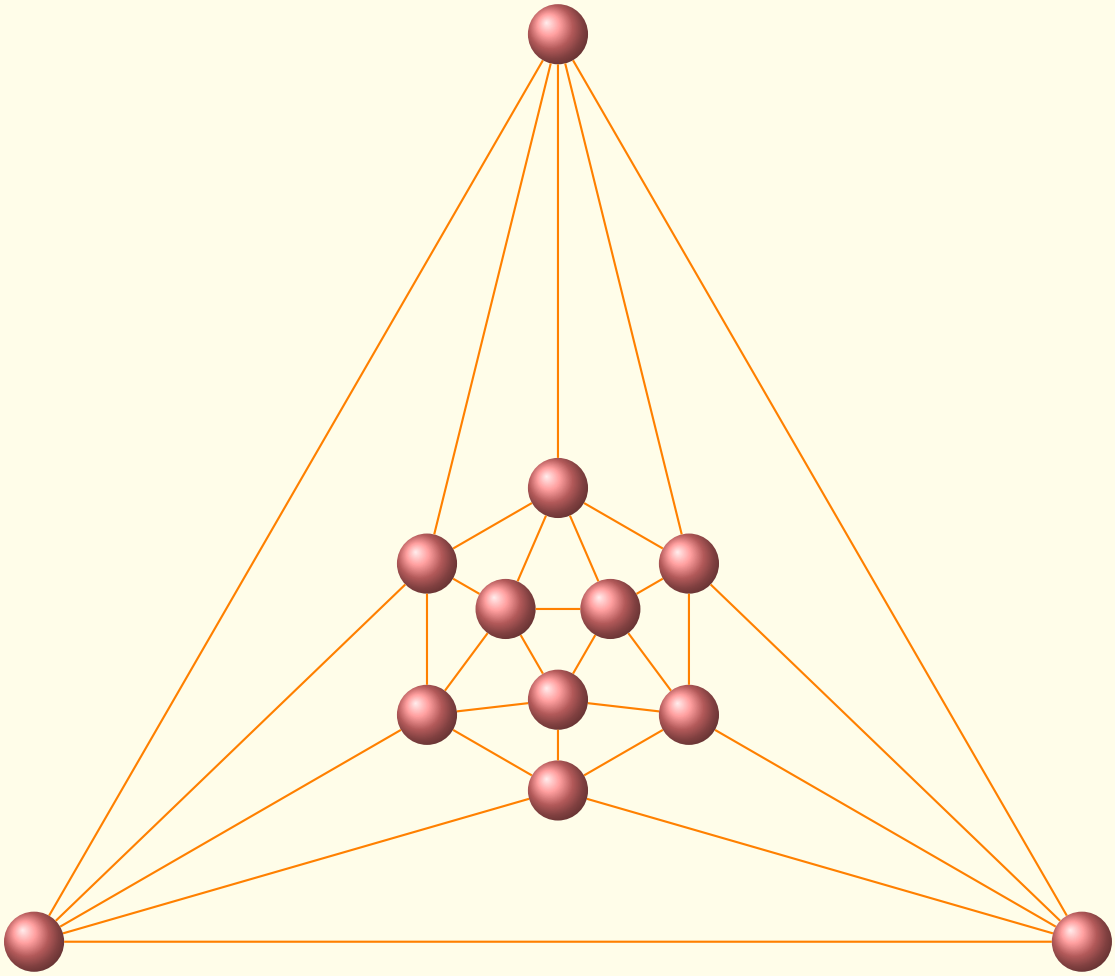
The Icosahedral Graph is 5-Regular.

26.7 Icosahedral forme 1



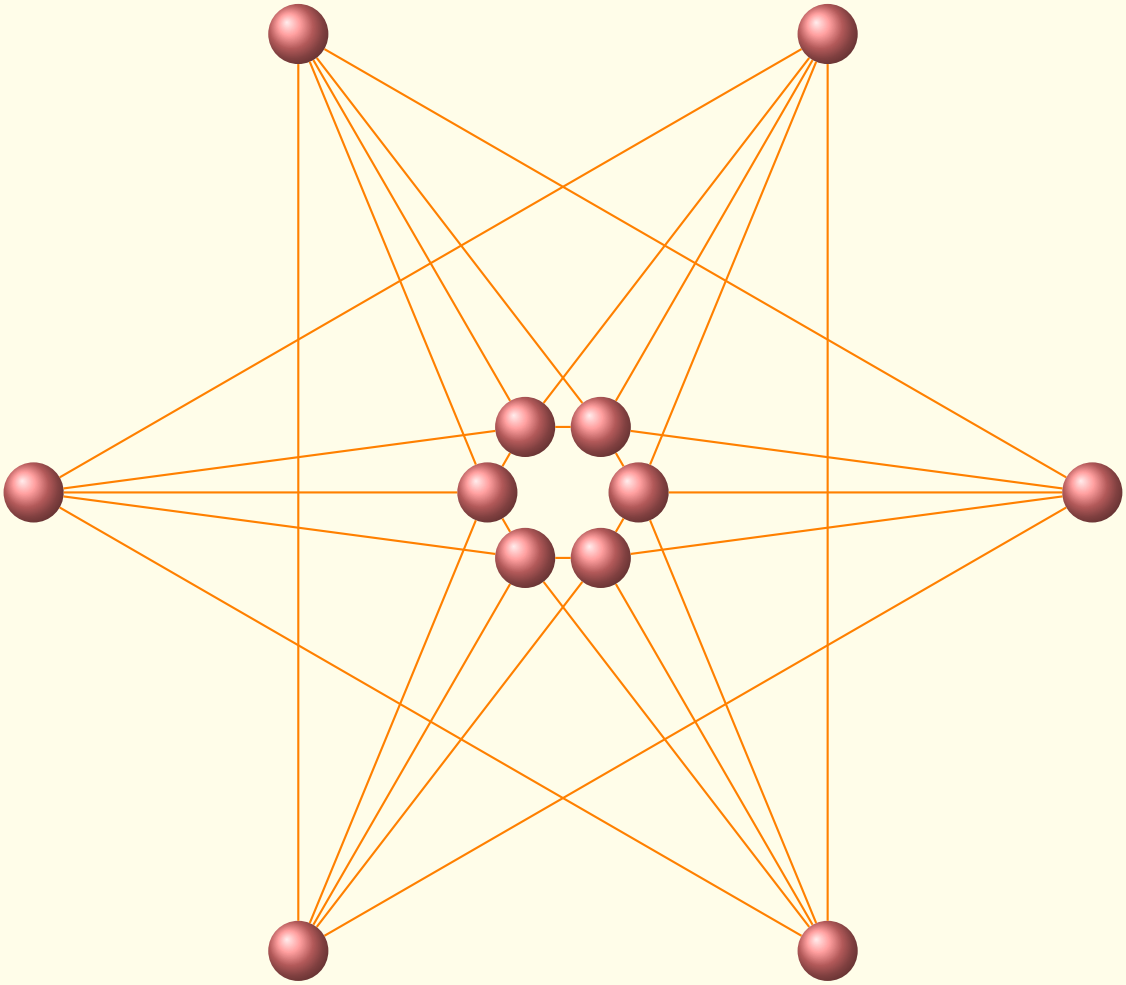
```
\begin{tikzpicture}[scale=.8]
 \GraphInit[vstyle=Art]\renewcommand*{\VertexInnerSep}{4pt}
 \SetGraphArtColor{red}{orange}
 \grIcosahedral[RA=5,RB=1]
\end{tikzpicture}
```

26.8 Icosahedral forme 2



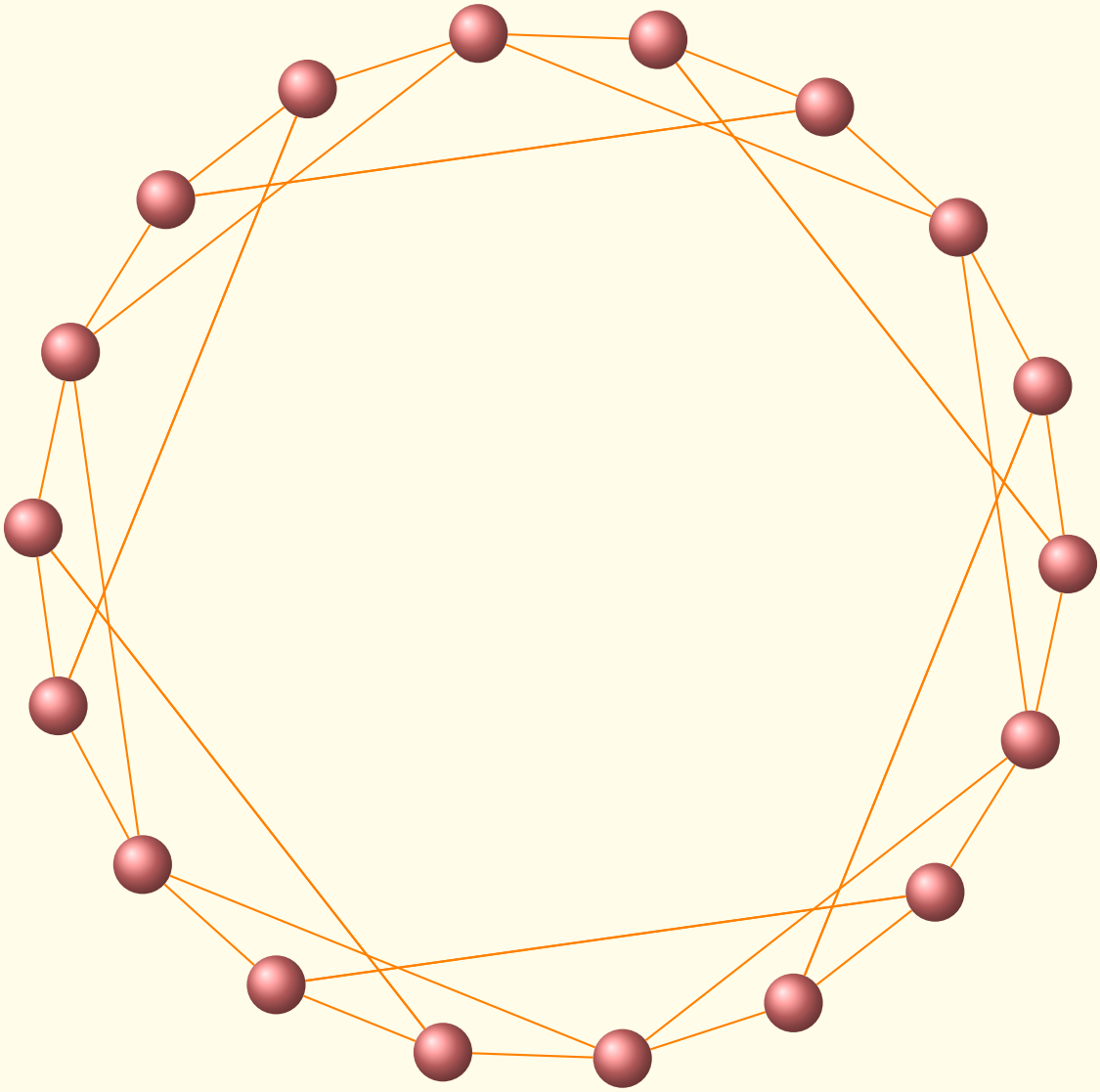
```
\begin{tikzpicture}[rotate=-30]
   \GraphInit[vstyle=Art]  \renewcommand*{\VertexInnerSep}{8pt}
   \SetGraphArtColor{red!50}{orange}
   \grIcosahedral[form=2,RA=8,RB=2,RC=.8]
\end{tikzpicture}
```

26.9 Icosahedral RA=1 et RB=7



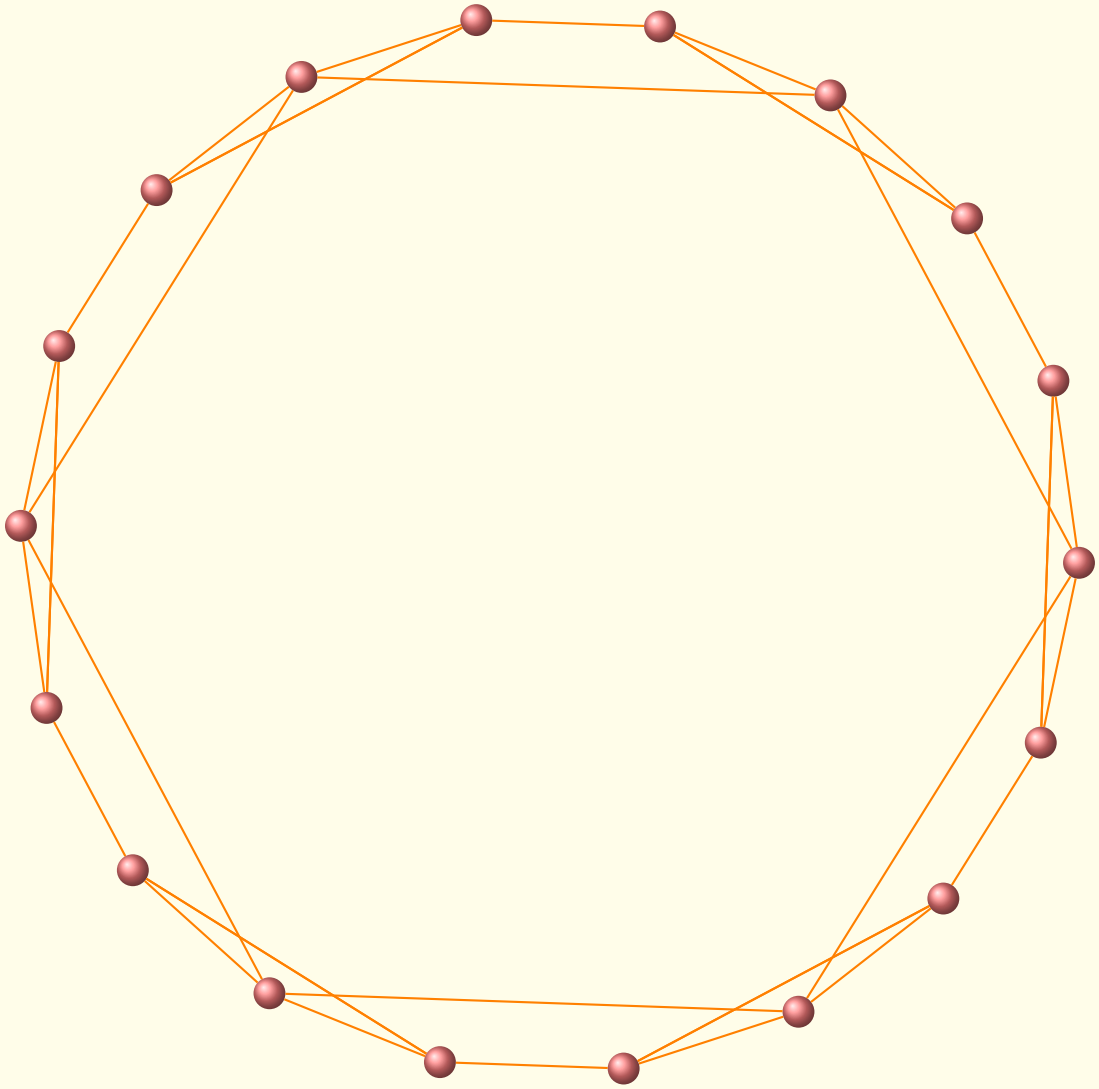
```
 \begin{tikzpicture}
    \GraphInit[vstyle=Art]  \renewcommand*{\VertexInnerSep}{8pt}
    \SetGraphArtColor{red!50}{orange}
    \grIcosahedral[RA=1,RB=7]
\end{tikzpicture}
```

26.10 Icosahedral LCF embedding 1



```
\begin{tikzpicture}[rotate=18]
    \GraphInit[vstyle=Art]  \renewcommand*{\VertexInnerSep}{8pt}
    \SetGraphArtColor{red!50}{orange}
    \grLCF[RA=7]{-4,-3,4}{6}
\end{tikzpicture}
```

26.11 Icosahedral LCF embedding 2



```
\begin{tikzpicture}[rotate=18]
   \GraphInit[vstyle=Art]
   \SetGraphArtColor{red!50}{orange}
   \grLCF[RA=7]{-2,2,3}{6}
\end{tikzpicture}
```

`\grDodecahedral[⟨RA=⟨Number⟩,RB=⟨Number⟩,RC=⟨Number⟩,RD=⟨Number⟩⟩]`

From MathWord : http://mathworld.wolfram.com/DodecahedralGraph.html

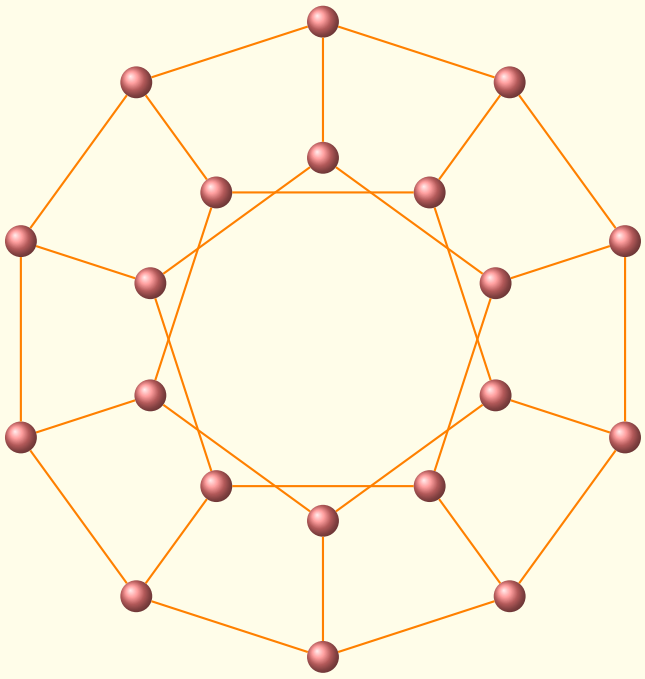
*The **Icosahedral Graph** is the Platonic graph corresponding to the connectivity of the vertices of a dodecahedron, illustrated above in four embeddings. The left embedding shows a stereographic projection of the dodecahedron, the second an orthographic projection, the third is from Read and Wilson, and the fourth is derived from LCF notation.* MathWorld by E.Weisstein

It has :

1. 20 nodes,
2. 30 edges,
3. graph diameter 5.

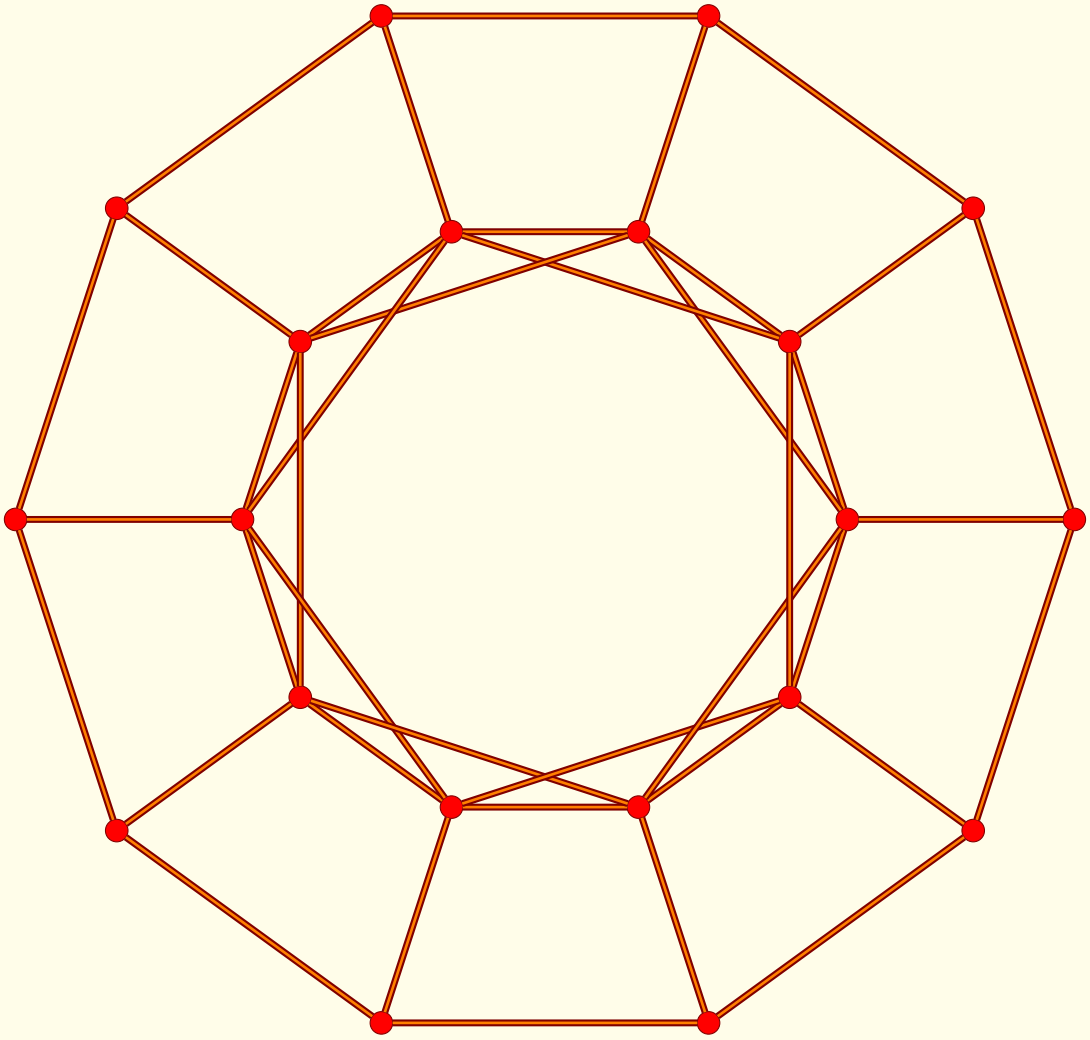
The Dodecahedral Graph is 3-Regular.

26.12 Dodecahedral



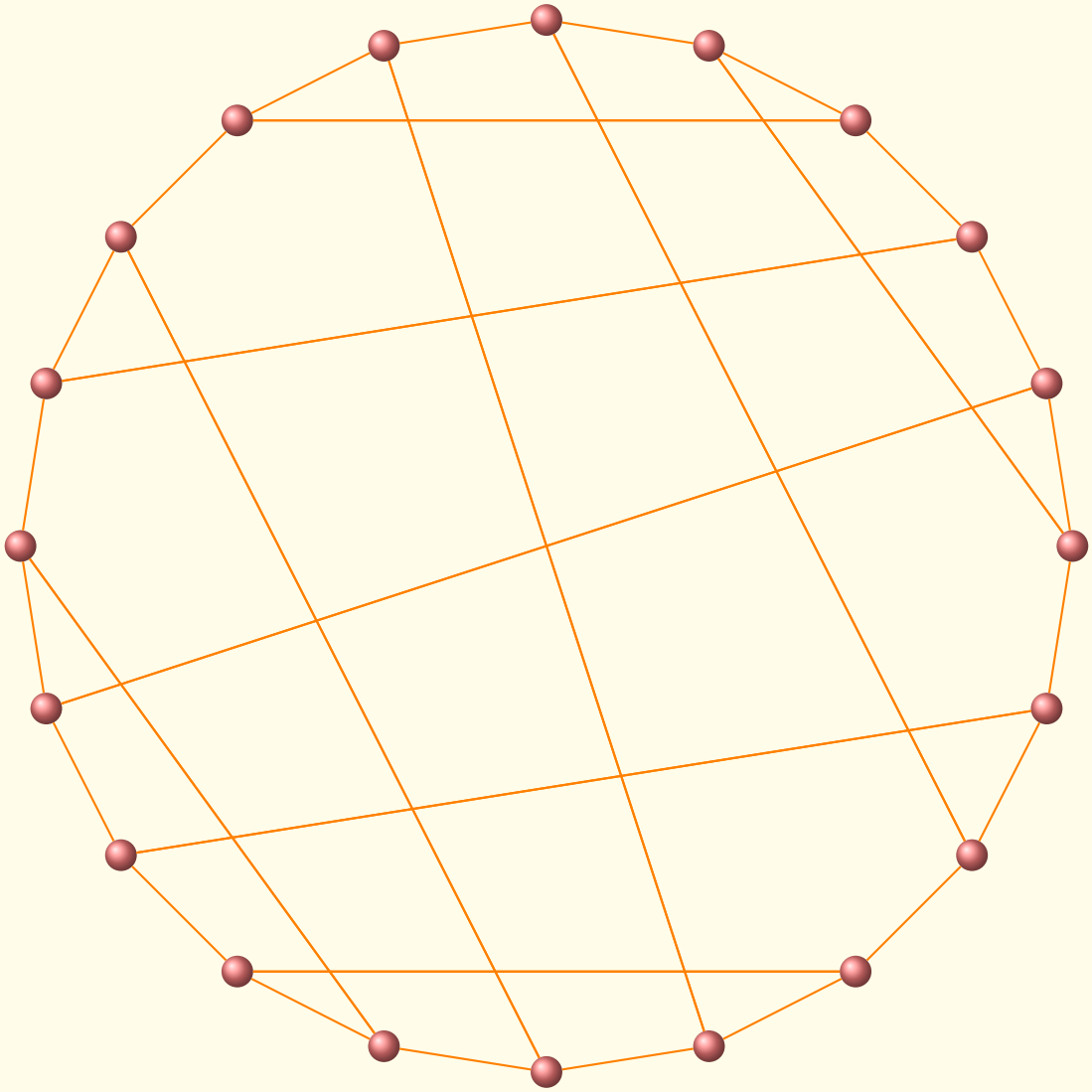
```
\begin{tikzpicture}[rotate=18,scale=.6]
   \GraphInit[vstyle=Art]
   \SetGraphArtColor{red!50}{orange}
   \grDodecahedral[RA=7,RB=4,RC=2,RD=1]
\end{tikzpicture}
```

26.13 Dodecahedral other embedding



```
\begin{tikzpicture}
   \grCycle[RA=7,prefix=a]{10}
   \grSQCycle[RA=4,prefix=b]{10}
     \foreach \v in {0,...,9}
       {\Edge(a\v)(b\v)}
\end{tikzpicture}
```

26.14 Dodecahedral LCF embedding



```
\begin{tikzpicture}[rotate=18]
   \GraphInit[vstyle=Art]
   \SetGraphArtColor{red!50}{orange}
   \grLCF[RA=7]{10,7,4,-4,-7,10,-4,7,-7,4}{2}
\end{tikzpicture}
```

SECTION 27

Robertson

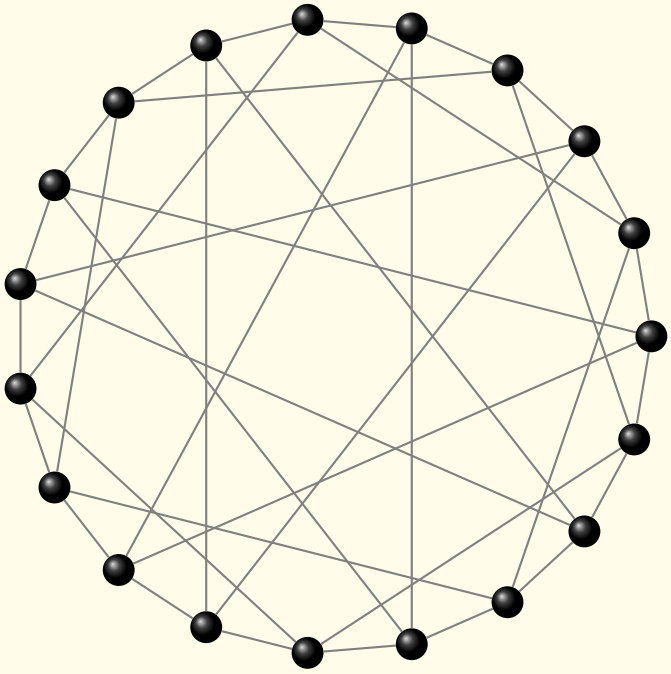
\grRobertson[⟨*options*⟩]{⟨*k*⟩}

From MathWord : http://mathworld.wolfram.com/RobertsonGraph.html

The Robertson graph is the unique (4,5)-cage graph, illustrated below. It has 19 vertices and 38 edges. It has girth 5, diameter 3, chromatic number 3, and is a quartic graph. MathWorld by E.Weisstein

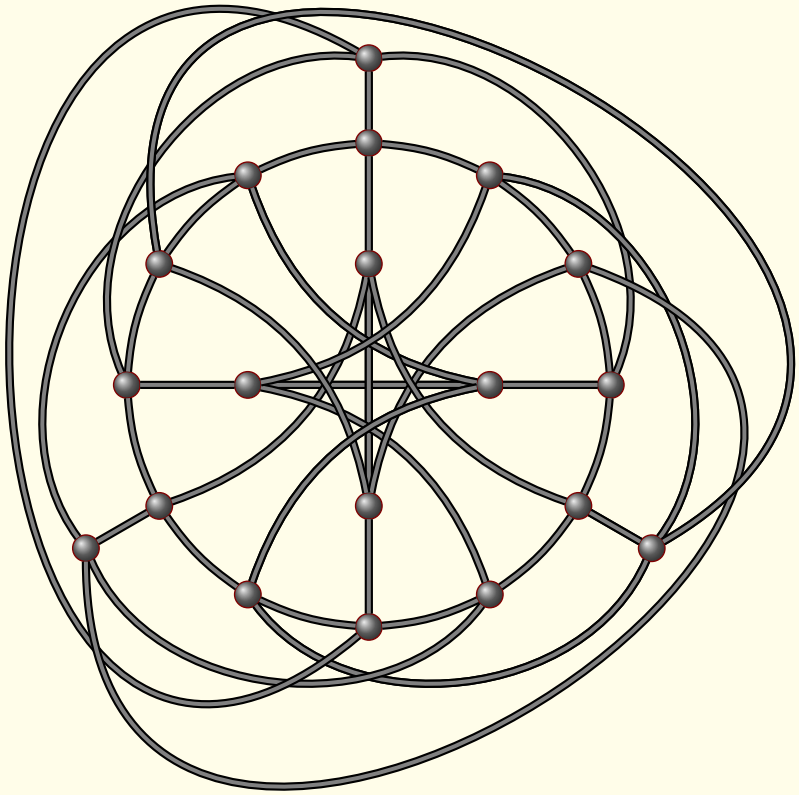
27.1 Robertson graph with \grRobertson

The cage



```
\begin{tikzpicture}[scale=.6]
  \GraphInit[vstyle=Art]
  \SetGraphArtColor{black}{gray}
  \grRobertson[RA=7]
\end{tikzpicture}
```

27.2 Fine embedding of the Robertson graph from RV



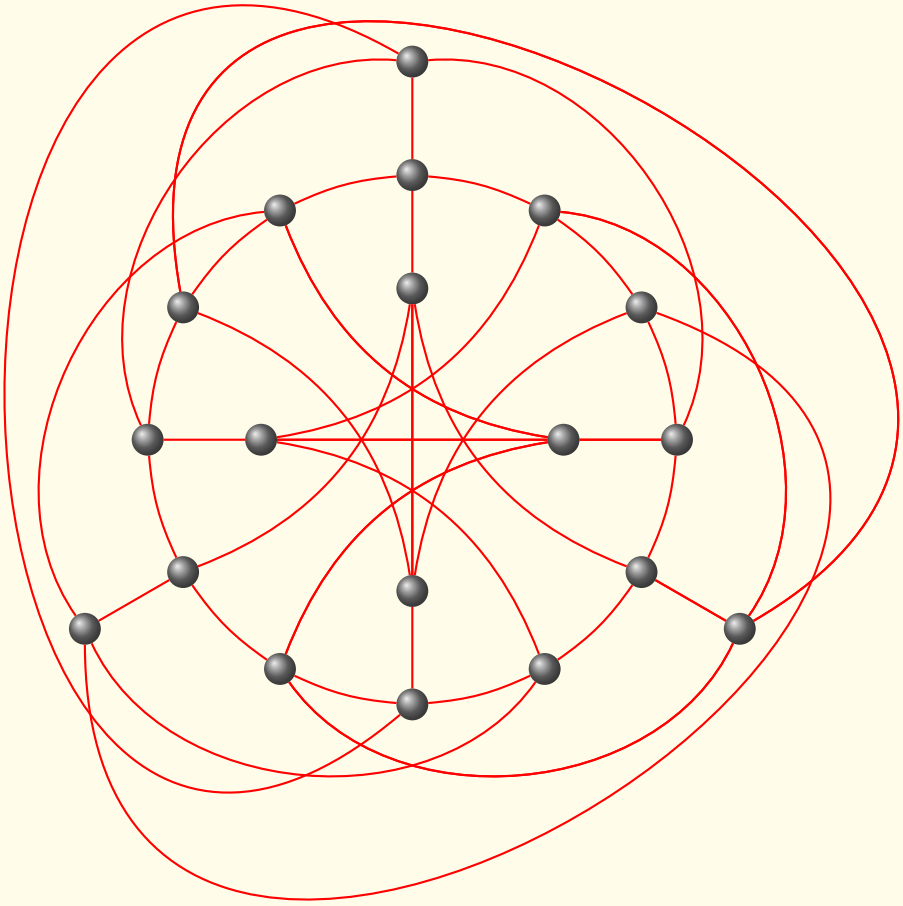
Code for the Robertson Graph

```
\begin{tikzpicture}[scale=.9]
  \tikzstyle{TempEdgeStyle}= [thick,black,%
                              double               = gray,%
                              double distance      = 1.5pt]%
   \SetVertexNoLabel
   \renewcommand*{\VertexBigMinSize}{14pt}
   \GraphInit[vstyle=Shade]
   \SetVertexNoLabel
   \SetUpEdge[style  = {thick,%
                        double          = orange,%
                        double distance = 1pt}]

   \SetGraphShadeColor{gray}{black}{gray}
   \tikzstyle{EdgeStyle} = [TempEdgeStyle]
   \begin{scope}[rotate=-30]
     \grEmptyCycle[RA=5.4]{3}
   \end{scope}
   \tikzstyle{EdgeStyle}= [TempEdgeStyle,bend right=10]
   \grCycle[prefix=b,RA=4]{12}
   \tikzstyle{EdgeStyle}= [TempEdgeStyle]
   \grCirculant[prefix=c,RA=2]{4}{2}
   \tikzstyle{EdgeStyle}= [TempEdgeStyle,bend left]
   \EdgeDoubleMod{c}{4}{0}{1}%
                 {b}{12}{4}{3}{4}
    \tikzstyle{EdgeStyle}= [TempEdgeStyle,bend right]
    \EdgeDoubleMod{c}{4}{0}{1}
                  {b}{12}{8}{3}{4}
    \tikzstyle{EdgeStyle}= [TempEdgeStyle]
    \EdgeDoubleMod{a}{3}{0}{1}%
                  {b}{12}{11}{4}{3}
    \EdgeDoubleMod{c}{4}{0}{1}%
                  {b}{12}{0}{3}{4}
    \tikzstyle{EdgeStyle}= [TempEdgeStyle,bend left=60]
    \EdgeDoubleMod{a}{3}{0}{1}%
                  {b}{12}{8}{4}{3}
    \tikzstyle{EdgeStyle}= [TempEdgeStyle,bend right=60]
    \EdgeDoubleMod{a}{3}{0}{1}%
                  {b}{12}{2}{4}{3}
    \tikzstyle{EdgeStyle}=[TempEdgeStyle,in=-50,out=-120,
                           relative,looseness=2.5]
    \EdgeDoubleMod{a}{3}{0}{1}%
                   {b}{12}{5}{4}{3}
\end{tikzpicture}
```

27.3 Robertson graph with new styles

The code with new styles, the result is on the next page.

```
  \begin{tikzpicture}[scale=1]
  \GraphInit[vstyle=Art]
  \SetGraphArtColor{gray}{red}
  \begin{scope}[rotate=-30]
    \grEmptyCycle[RA=5]{3}
  \end{scope}
  {\tikzset{EdgeStyle/.append style = {bend right=10}}
  \grCycle[prefix=b,RA=3.5]{12}}
  \grCirculant[prefix=c,RA=2]{4}{2}
  {\tikzset{EdgeStyle/.append style = {bend left}}
  \EdgeDoubleMod{c}{4}{0}{1}%
                {b}{12}{4}{3}{4}}
  {\tikzset{EdgeStyle/.append style = {bend right}}
   \EdgeDoubleMod{c}{4}{0}{1}
                 {b}{12}{8}{3}{4}}
   \EdgeDoubleMod{a}{3}{0}{1}%
                 {b}{12}{11}{4}{3}
   \EdgeDoubleMod{c}{4}{0}{1}%
                 {b}{12}{0}{3}{4}
  {\tikzset{EdgeStyle/.append style = {bend left=60}}
   \EdgeDoubleMod{a}{3}{0}{1}%
                 {b}{12}{8}{4}{3}}
  {\tikzset{EdgeStyle/.append style = {bend right=60}}
   \EdgeDoubleMod{a}{3}{0}{1}%
                 {b}{12}{2}{4}{3}}
   {\tikzset{EdgeStyle/.append style = {in=-50,out=-120,%
                                        relative,looseness=2.5}}
   \EdgeDoubleMod{a}{3}{0}{1}%
                  {b}{12}{5}{4}{3}}
\end{tikzpicture}
```

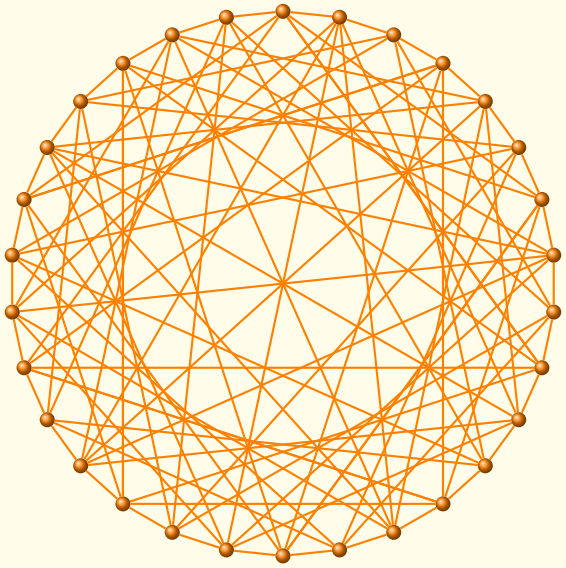


\grRobertsonWegner[⟨*options*⟩]{⟨*k*⟩}

From MathWord : http://mathworld.wolfram.com/Robertson-WegnerGraph.html

he Robertson-Wegner graph is of the four (5,5)-cage graphs, also called Robertson's cage . Like the other (5,5)-cages, the Robertson-Wegner graph has 30 nodes. It has 75 edges, girth 5, diameter 3, and chromatic number 4.
MathWorld by E.Weisstein

27.4 Robertson-Wegner graph



```
\begin{tikzpicture}[rotate=90,scale=.6]
    \GraphInit[vstyle=Art]
    \tikzset{VertexStyle/.append style={minimum size=2pt}}
    \grRobertsonWegner[RA=6]
\end{tikzpicture}
```

The next code gives the same result

```
\begin{tikzpicture}[rotate=90]
    \GraphInit[vstyle=Art]
    \grLCF[RA=6]{6,12}{15}
    \EdgeInGraphMod{a}{30}{9}{1}{6} \EdgeInGraphMod*{a}{30}{15}{2}{6}
    \EdgeInGraphMod*{a}{30}{9}{3}{6}
\end{tikzpicture}
```

SECTION 28

Tutte-Coxeter

\grTutteCoxeter[⟨*options*⟩]

From MathWord : http://mathworld.wolfram.com/LeviGraph.html

The Levi graph is the unique (3,8)-cage graph and Moore graph. It is also distance-regular and is also called the Tutte-Coxeter graph or Tutte's 8-cage.

MathWorld by E.Weisstein

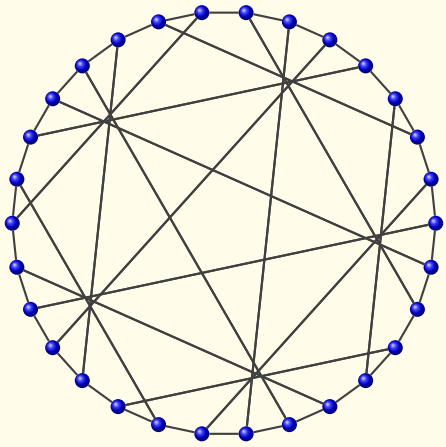
From Wikipedia : http://en.wikipedia.org/wiki/Tutte\T1\textendashCoxeter_graph

In the mathematical field of graph theory, the Tutte–Coxeter graph or Tutte eight-cage is a 3-regular graph with 30 vertices and 45 edges. As the unique smallest cubic graph of girth 8 it is a cage and a Moore graph. It is bipartite, and can be constructed as the Levi graph of the generalized quadrangle. The graph is named after William Thomas Tutte and H. S. M. Coxeter; it was discovered by Tutte (1947) but its connection to geometric configurations was investigated by both authors in a pair of jointly published papers (Tutte 1958; Coxeter 1958a).

28.1 Tutte-Coxeter graph (3,8)-cage or Levi graph

An other method to get the same result is :

```
\grLCF[RA=7]{-13,-9,7,-7,9,13}{5}
```



```
\begin{tikzpicture}[scale=.7]
   \GraphInit[vstyle=Art]
   \tikzset{VertexStyle/.append style={minimum size=2pt}}
   \SetGraphArtColor{blue}{darkgray}
   \grTutteCoxeter
\end{tikzpicture}
```

SECTION 29

Wong

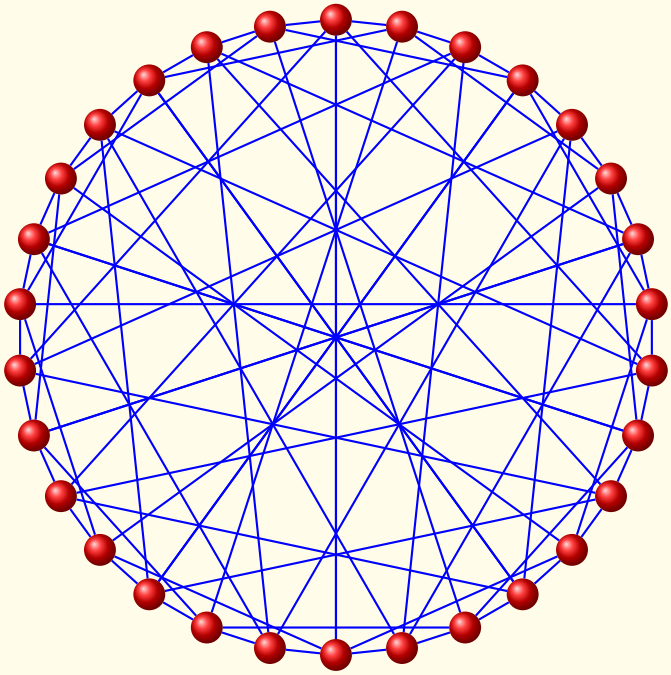
\grWong[⟨*options*⟩]

From MathWord : http://mathworld.wolfram.com/WongGraph.html

The Wong graph is one of the four $(5,5)$*-cage graphs. Like the other -cages, the Wong graph has 30 nodes. It has 75 edges, girth 5, diameter 3, chromatic number 4.* MathWorld by E.Weisstein

29.1 Wong graph

You can see the cage definition here : 5



```
\begin{tikzpicture}[rotate=90,scale=.6]
   \GraphInit[vstyle=Art]
   \SetGraphArtColor{red}{blue}
   \grWong[RA=7]
\end{tikzpicture}
```

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